

On the Complexity and Accuracy of Geographic Profiling Strategies

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Geographic profilers have access to a repertoire of strategies for predicting a serial offender's home location. These strategies range in complexity—some involve more calculations to implement than others—and the assumption often made is that more complex strategies will outperform simpler strategies. In the present study, we tested the relationship between the complexity and accuracy of 11 strategies. Data were crime site and home locations of 16 UK residential burglars who had committed 10 or more crimes each. The results indicated that strategy complexity was not positively related to accuracy. This was also found to be the case across tasks that ranged in complexity (where complexity was determined by the number of crimes used to make a prediction). Implications for police policies and procedures, as well as our understanding of human decision-making, are discussed.

KEY WORDS: geographic profiling; complexity; accuracy; serial burglary; *CrimeStat*, decision-making.

1. INTRODUCTION

An implicit assumption in decision-making research is that more complex decision-making strategies lead to more accurate predictions (Brehmer, 1994; Hammond, 1990; Hogarth, 1980; Kahneman and Tversky, 1973). Complex strategies make more calculations when presented with information, and because they take into account extra information and appear to evaluate it in a more sophisticated fashion, they are often assumed

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to be more accurate than simpler strategies (Payne *et al.*, 1988, 1993). This assumption has led many researchers to devote considerable time to developing more complex strategies in a range of contexts (Ashenfelter *et al.*, 1995; Carter and Polger, 1986; Cronbach and Gleser, 1965), and many of these are now routinely implemented in actuarial tools (Swets *et al.*, 2000).

Recently, this “complexity equals accuracy” assumption has been challenged. By decomposing decision-making strategies into the number of steps required to process a given amount of information, Payne *et al.* (1988, 1993) showed that simpler strategies (i.e., those that require fewer computational steps) can sometimes yield levels of accuracy similar to those produced by more complex strategies. Research has also shown that simple strategies do not necessarily lead to a reduction in accuracy as problems become more complex (Paquette and Kida, 1988; Payne *et al.*, 1993). For example, Gigerenzer *et al.* (1999) showed that individuals were able to use heuristics to reduce complex problems into simpler judgments and still perform as accurately as actuarial techniques. These initial findings have led Gigerenzer and colleagues to question the need for complex strategies in a number of decision-making contexts, particularly where the time and cost of making decisions can have serious consequences (e.g., emergency medical situations).

One criminological context where this complexity–accuracy assumption is being challenged is in the area of geographic profiling (Snook *et al.*, 2002, 2004). In its most basic form, the geographic profiling task requires a prediction of where a serial offender is most likely to be residing based on where he has committed his crimes (Canter *et al.*, 2000; Rossmo, 2000). The most common way of making these predictions is to use mathematical functions, which are typically incorporated into computerized geographic profiling systems (Canter *et al.*, 2000; Rossmo, 2000; Taylor *et al.*, 2002). Yet, despite the use of geographic profiling systems, simpler strategies are available to make these predictions and some evidence suggests that they can be as accurate. For example, Snook *et al.* (2002) demonstrated that individuals introduced to two simple patterns of criminal spatial behavior were able to achieve a level of accuracy in a geographic profiling task comparable to that achieved by a geographic profiling system. These findings were replicated by Snook *et al.* (2004), who further showed not only that teaching one strategy was sufficient to reach performance comparable to a complex strategy, but also that some individuals were using simple strategies to make accurate predictions prior to training. In both of these studies, however, comparisons were only made against one complex strategy, such that there has been no systematic study of strategy complexity and its relationship to accuracy. Thus, an important question remains about the complexity–accuracy

assumption in this domain: Do complex strategies result in more accurate geographic profiling predictions compared to simpler strategies?

1.1. A Repertoire of Geographic Profiling Strategies

Geographic profilers have access to a range of strategies for predicting where a serial offender is residing. These are described succinctly by Levine and Associates (2000), who make the broad distinction between *spatial distribution strategies* and *probability distance strategies*. Spatial distribution strategies include a number of different procedures, all of which predict the home location of a serial offender by calculating a central point from a distribution of crime site locations. Some common spatial distribution strategies include the *center of the circle*, *centroid*, *median*, *geometric mean*, *harmonic mean*, and *center of minimum distance* (these are defined in Section 2.3.1 and the Appendix A).

Probability distance strategies begin with the assumption that an offender's crime site locations define their activity space, and that this area contains the offender's residence (Canter *et al.*, 2000; Rossmo, 2000). The prediction of an offender's residence within this space is achieved by applying, around each crime site, a mathematical function that assigns areas, or cells, of the space a small positive real number. The numbers produced by applying the function with respect to each of the crime sites are added up to produce an overall value that is then associated with each cell of the space. The result is a surface that indicates the likelihood of an offender living at a particular location, making it possible to locate the area that is most likely to contain the offender's residence. Probability distance strategies differ from one another in terms of the shape of the mathematical function applied around each crime site and the assumptions regarding the relationship between where offenders reside and where they commit their offences. Common probability distance functions include the *negative exponential*, *normal*, *lognormal*, *linear* and *truncated negative exponential* (these are defined in Section 2.3.2 and in Appendix A).

1.2. Known Differences in the Accuracy of Geographical Profiling Strategies

Since no attempt has been made to define the complexity of the geographic profiling strategies outlined above, it is prudent to first question whether they differ in their level of accuracy, while disregarding their complexity. If the assumed relationship between complexity and accuracy exists, then we would first expect to find some strategies performing with more accuracy than other strategies. These strategies may then be predicted

as being more complex, and the complexity–accuracy assumption can be systematically assessed.

Unfortunately, while the question of strategy accuracy has received attention in the literature, it is difficult to draw conclusions regarding relative strategy performance because accuracy has been assessed by different measures. For example, Canter and Larkin (1993) examined the accuracy of a circle strategy, which states that a serial offender's home will be located within a circle with its diameter defined by the distance between that offender's two furthest offences. They found that 87% of serial rapists from the UK had their homes located within their circle. Although subsequent research in Australia (Kocsis and Irwin, 1997) has supported this finding for serial rapists (71% found within the circle) and arsonists (82%) lower percentages have been reported for US serial rapists (56%), Australian burglars (48%) and Japanese arsonists (51%) (see Kocsis and Irwin, 1997; Tamura and Suzuki, 1997; Warren *et al.*, 1998).

In a study testing a family of negative exponential functions on 70 US serial killers, Canter *et al.* (2000) measured accuracy using *search cost*, which is the percentage of cells in an overlaid grid that need to be searched to locate the cell that contains an offender's home. They found that a number of different functions were good at predicting home location, with the best parameters resulting in an average search cost of 11%. In other words, on average across their sample, 11% of an offender's activity space (i.e., total search area) had to be searched before their home was found. Since a circle drawn around the two furthest crimes is likely to incorporate more than 11% of the total search area, this result might tentatively suggest that probability based strategies outperform the circle strategy.

Similarly, in testing the accuracy of a criminal geographic targeting algorithm (CGT) on 15 serial killers, Rossmo (2000) quantified accuracy using a measure equivalent to search cost (*hit percentage*) and found an average hit percentage of 6%. Although that lower value suggests that the CGT strategy is more accurate than the negative exponential function used by Canter *et al.* (2000), Rossmo was more cautious in selecting his original sample, choosing offences that were appropriate for the assumptions of the strategy being tested. In particular, Rossmo asserted that probability strategies require information about the location of at least five crime sites to increase reliability, a criterion not used by Canter and his colleagues. More specifically, he argued that strategy accuracy increased as the number of crimes used to make a prediction increased. Other evidence to support this suggestion comes from Levine and Associates (2000), who report improvements in the mean accuracy of a number of strategies when implemented on the locations of three to five crimes, six to nine crimes, and 10 or more crimes.

Finally, in the only existing comparison of many strategies, Levine and Associates (2000) compared the accuracy of six spatial distribution strategies and four probability strategies. For each strategy, a prediction was made for 50 different serial offenders and accuracy was calculated by measuring (in miles) the straight-line distance between the predicted and actual home location. The mean accuracies across the 10 strategies suggested little difference in the performance of each strategy, although no statistical tests were conducted to confirm this observation.

In sum, existing research is divided on the issue of whether certain geographic profiling strategies perform better than other strategies. Furthermore, the complexity of the various geographic profiling strategies that exist for this purpose has never been quantitatively defined, and no standard measure of accuracy has been adopted across the studies that have been conducted.

1.3. Hypotheses

The following two hypotheses were tested:

1. Complex geographic profiling strategies will result in more accurate predictions compared to less complex strategies.
2. Complex geographic profiling strategies will be more accurate than less complex strategies on more complex tasks.

2. METHOD

2.1. The Sample

Data were crime site and home locations of 16 serial residential burglars who committed 10 or more burglaries in a semi-rural county of the UK between 1997 and 1999. We define a serial residential burglary as any offender that was arrested for 3 or more residential burglaries. The average number of crimes committed by the burglars was 20 ($SD = 8.9$). Ten of the burglars had one home location, four had two home locations, and two had three home locations. Since some of the burglars had multiple home locations (i.e., they changed residential locations at some point during their series), we decided that predictive accuracy should be measured for all home locations, thus, there were 24 cases where serial offender home locations could be predicted. The average home to crime distance measured in kilometers for cases involving 5, 6, 7, 8, 9 and 10 crimes were, respectively, 8.9 ($SD = 15.8$), 8.8 ($SD = 15.7$), 8.7 ($SD = 15.6$), 8.8 ($SD = 15.7$), 8.0 ($SD = 14.8$),

and 8.0 (SD = 14.8). The area of a circle (km²) drawn around the two most distant crimes were, for the 5, 6, 7, 8, 9 and 10-crime series respectively, 31.2 (SD = 46.9), 36.0 (SD = 46.4), 40.0 (SD = 48.8), 40.0 (SD = 48.8), 42.6 (SD = 49.1), and 46.7 (SD = 53.4).

2.2. The Geographic Profiling Strategies

Eleven geographic profiling strategies were tested. Six of the strategies were spatial distribution strategies including the center of the circle, centroid, median, geometric mean, harmonic mean, and center of minimum distance. The remaining five strategies were probability distance strategies including functions that are linear, negative exponential, normal, lognormal, and truncated negative exponential. These probability strategies, or variations of these strategies, are implemented as algorithms in popular geographic profiling systems such as *Dragnet* (Canter *et al.*, 2000), *CrimeStat II* (Levine and Associates, 2000), and *Rigel* (Rossmo, 2000).

2.3. Defining the Complexity of each Geographic Profiling Strategy

Computer scientists and decision-making researchers have used similar methods for defining the complexity of problem solving strategies. We drew on their approaches to study the complexity of geographical profiling strategies.

In decision-making research, complexity is often measured by counting the number of mathematical operations required to transform an initial state (e.g., distribution of crimes) into a final state (e.g., predicted home location) (e.g., Newell and Simon, 1972; Payne *et al.*, 1988). Mathematical operations, or elementary information processors, include adding, subtracting, dividing, multiplying, and so on. As Payne *et al.* (1988) note, counting the number of mathematical operations provides a common language that allows for comparisons across disparate prediction strategies, in addition to providing a method that is reasonable for approximating complexity.

Similarly, in computer science, complexity has become an important area of research initiated formally by Hartmanis and Stearns (1965), who built on work dating back to Turing (1936). They defined a framework by which it is possible to characterize mathematically the complexity of problems that can be solved on a computer. Researchers in this area seek the definition of a mathematical law that describes how the running time (or indeed any other measurable resource used during a computation) of a

given computer program varies as a function of the size of the problem that is being solved. The *computational* complexity of a computer program is defined in terms of the maximum number of computation steps needed to run a particular program on a computer on any input task of a given size. The complexity of a problem can then be defined in terms of the best possible program that solves it (e.g., Bovet and Crescenzi, 1994).

One of the aims of this paper is to classify different geographic profiling strategies according to some measure of complexity. We believe that the computational complexity approach described above is well suited for such analysis for the following reasons:

- (a) all the profiling strategies considered in this paper are relatively simple and lend themselves to being encoded in a computer program;
- (b) the mathematically precise setting of computational complexity offers techniques for deriving tight estimates on the complexity of the various profiling methods.

In the forthcoming sections, we will define two profiling strategies and provide, in each case, the description of a computer program implementing the given strategy. By means of such implementations it is possible to give an upper bound on the strategy's complexity. Because these upper bounds will be calculated from structural properties of the various strategies, their relative values will remain invariant across different computer systems or programming approaches.

All programs described in this paper can be implemented quite easily in the reader's favorite programming language. However, following Aho *et al.* (1974) or Cormen *et al.* (2001), we describe the implementation of the various strategies using either a simple *pseudo*-programming language or plain English. We follow the conventions of Cormen *et al.* (2001) (see Section 2.1 of their book) on the computational device on which our programs would run, and on the data types and instruction set we are allowed to use.

For the purpose of estimating each program's complexity, we assume that:

- (i) Elementary arithmetic operations such as $+$, \times , $-$, and \div can be computed in one time step; we also assume that \sqrt{x} and the transcendental function e^x , and its inverse $\log x$ can be computed in a constant number of steps (e.g., see Tang and Tak, 1989). The particular method used to implement these functions does not affect the quality of our results if the lines in Fig. 1 (see Section 3.1) are plotted under the assumption that all arithmetic operations are equal to one time step.
- (ii) Elementary instructions in any program always have the form: “do something to some variable(s) and store the result in some variable”.

The “do something to some variable(s)” component involves computing some arithmetical expression involving the elementary arithmetic operations described above. The number of steps needed to complete such elementary assignment instructions was calculated as the total number of steps needed to compute each of its elementary arithmetic operations plus one. So, for instance, “store the value 25 in the variable D” takes just one step (no arithmetic operation involved), while “take the value of x , add 25 to it, and store the result in the variable D” takes two steps because of the addition operation.

(iii) Selection constructs of the form:

“**if** (some condition) do something, **else do** something else”

take a number of steps given by the maximum between the number of steps needed for “do something” and “do something else” (we assumed that conditions, always involving a relational operator like = or \leq applied to two variables, cost nothing).⁶

(iv) Loops are always of the form

“**for** i = starting value, **do** something **until** i becomes larger than “final value”

(at which point execution of the loop is terminated). We assume such a structure requires one initial step to set the value of i to “starting value”, then as many steps as needed to perform “do something” plus one to increase i , repeated by the number of times the whole cycle is repeated.

(v) Elementary assignment instructions, selections constructs, and loops can be combined together in sequences. The total running time of a program is the sum of the running times of its components.

We further assume that the *input* is present inside the computer memory as a sequence of pairs, $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, where each pair is formed by two non-negative integer values representing the x and y co-ordinates of a given crime site location. The *size* of the input (or task complexity) is defined as the total number of points for a given crime series. The *output* was an estimated home location given as an x and y co-ordinate.

2.3.1. Spatial Distribution Strategies

In this section, we provide an example of a complexity analysis for a spatial distribution strategy. The complexity analysis of the remaining five

⁶It could be argued that relational operators cost one unit of time, which would result in minor changes being made in all complexity calculations. However, no major difference in the relative complexity of the various strategies would be observed.

spatial distribution strategies is presented in Table I and the program used to calculate the complexity of the strategy is provided in Appendix A. In all cases, the computational complexity of implementing such strategies is at least linear in the number of crime locations. In each case, $T(n)$ denotes the computational complexity of the particular method.

Given n points, the *center of the circle* is calculated as the mid-point of the two furthest points in the sequence of crime locations. The following program computes the center of the circle. All variables only take integer values, except DIST and d , which may be any real number.

```

set  $i_{\max}$  and  $j_{\max}$  to one;
set  $d$  to zero;
for  $i = 1$  to  $n - 1$ 
  for  $j = i + 1$  to  $n$ 
    set DIST to the Euclidean distance between  $(x_i, y_i)$  and  $(x_j, y_j)$ ;
    if DIST  $>$   $d$ 
      set  $d$  to DIST;
      set  $i_{\max}$  to  $i$ ;
      set  $j_{\max}$  to  $j$ ;

```

return estimated home location as mid-point between $(x_{i_{\max}}, y_{i_{\max}})$ and $(x_{j_{\max}}, y_{j_{\max}})$

The first two instructions cost three units of time (simple assignment instructions). If T is the time to run (once) all the instructions inside the two nested loops then the overall complexity of the two nested loops is:

$$(n - 1) + (T + 1) \times [(n - 1) + (n - 2) + \dots + 1] = (n - 1) + (T + 1) \times \left(\frac{n}{2}\right), \quad (1)$$

where the first $n - 1$ steps are for the assignments to i and each term in the square brackets gives the number of iterations of the inner loop on j for each iteration of the outer loop. We compute the distance between point (x_i, y_i) and point (x_j, y_j) using the formula:

$$\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \quad (2)$$

with two subtractions, two multiplications, and a square root operation. If ROOT_2 is the cost of taking the square root of a number, then:

$$T \leq 4 + \text{ROOT}_2 + 3. \quad (3)$$

Table I. Complexity Analysis

Geographic profiling strategy	Formula	Complexity formula
1. Centroid Point whose coordinates are the mean of the x and y -coordinates of the crime sites	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i,$	$T(n) \leq 2(2n + 1)$
2. Harmonic mean Point whose coordinates are the inverse mean of the inverse coordinates	$\bar{x} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}, \bar{y} = \frac{n}{\sum_{i=1}^n \frac{1}{y_i}}$	$T(n) \leq 2(3n + 1)$
3. Geometric mean The anti-log of the mean of the logarithms of the coordinates	$\bar{x} = e^{\frac{1}{n} \sum_{i=1}^n \log x_i}, \bar{y} = e^{\frac{1}{n} \sum_{i=1}^n \log y_i}$	$T(n) \leq 2(n \times (\text{LOG} + 2) + \text{EXP} + 1)$
4. Median The middle value of the distribution of co-ordinates	$\bar{x} = \frac{1}{2} \left(x_{\lfloor \frac{n}{2} \rfloor} + x_{\lfloor \frac{n}{2} \rfloor + 1} \right), \bar{y} = \frac{1}{2} \left(y_{\lfloor \frac{n}{2} \rfloor} + y_{\lfloor \frac{n}{2} \rfloor + 1} \right)$	$T(n) \leq 4n^2 + 6n + 10$
5. Center of the circle The mid-point of the two furthest points in the sequence of crime locations.	See description in Section 2.	$T(n) \leq 2\{4n^2 - 3n + 3\}$
6. Center of minimum distance The point in a grid where the sum of the distance between that point and all crime locations is smallest	$W(\bar{x}, \bar{y}) = \sum_{i=1}^n \text{dist}((x_i, y_i), (\bar{x}, \bar{y}))$	$T(n) \leq 14((9 + \text{ROOT}_2)n + 2)$
7. Linear The probability of an offender living at a particular location decreases in a linear fashion with increasing distance away from a crime site.	See description in Section 2.	$T_f = 2$

<p>8. Negative exponential The probability of an offender living at a particular location decreases exponentially with increasing distance away from a crime site.</p>	$f(x) = ae^{-bx}$ with $a, b > 0$	$T_f = 2 + \text{EXP}$
<p>9. Truncated negative exponential A spline function consisting of the linear strategy for $x \leq x_0$ and the negative exponential strategy</p>	$f(x) = \begin{cases} bx & \text{if } x \leq x_0 \\ bx_0 e^{-c(x-x_0)} & \text{if } x > x_0 \end{cases}$	$T_f = 4 + \text{EXP}$
<p>10. Normal The function rises to a peak likelihood (distance) and then declines</p>	$f(x) = \frac{ae^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma}}$	$T_f = 7 + \text{EXP} + \text{SQRT}_2$
<p>11. Lognormal Similar to the normal strategy but positive or negatively skewed. Notice that this function really depends on the square of x. Positive parameters a, μ, and σ are normally taken to be one, and generally input by the user.</p>	$f(x) = \frac{a^{2\log(x-\mu)^2}}{x^2 \sqrt{2\pi\sigma}}$	$T_f = 10 + \text{EXP} + \text{SQRT}_2 + \text{LOG}$

The total cost of this method is, therefore:

$$T(n) \leq \left\{ (n-1) + (\text{ROOT}_2 + 8) \times \left(\frac{n}{2}\right) + 4 \right\}, \quad (4)$$

where $\left(\frac{n}{2}\right) = \frac{n(n-1)}{2}$.

The analysis presented above can in fact be improved. The function $f(x) = \sqrt{x}$ is strictly monotone increasing for each $x \geq 0$. Hence, we obtain a slightly faster, but equivalent, program by replacing the Euclidean distance with its squared value (assuming the numbers under consideration are not too large). The complexity of this modified program is:

$$T(n) \leq 2 \left\{ (n-1) + 8 \times \left(\frac{n}{2}\right) + 4 \right\} = 2 \{ 4n^2 - 3n + 3 \}. \quad (5)$$

Of note is that this result is very close to best possible. Although in general $T(n) \geq n$ in this situation any program computing the center of the circle of a set of n points will need to compute $\left(\frac{n}{2}\right)$ distances. Hence, a more accurate lower bound on this method's complexity is $T(n) \geq \left(\frac{n}{2}\right)$.

2.3.2. Probability Distance Strategies

As mentioned, probability distance strategies assume that crime site locations represent an offender's activity space, and that this space contains that offender's residence (Taylor *et al.*, 2002). Given the n input points (i.e., crime locations), a grid is laid on top of them. We let (u_1, v_1) , (u_2, v_2) , \dots , (u_m, v_m) be the centers of the various cells defined by such a grid. For each center, $C_i = (u_i, v_i)$, we then define the Euclidean distance, $\text{dist}_{i,j}$, from C_i of the j th crime location, (x_j, y_j) (n values defined for each center). Then, a given probability density function, f , is evaluated and the numerical value associated to C_i is:

$$l(C_i) = \sum_{j=1}^n f(\text{dist}_{i,j}). \quad (6)$$

The predicted home location is then set to be the center, C , that maximizes function l . In other words, a numerical score is assigned to a number of points in the region where the offences were committed, allowing us to identify the point with the highest probability score.

For each of the probability distance strategies, we used a common formula to measure complexity. Calling T_f the time to evaluate f , and assuming that m centers are defined, each of the methods in this section had an approximate complexity of at most:

$$m \times n \times T_f \quad (7)$$

computational steps. Clearly, if m is much smaller than n , these methods can be simpler than many of the spatial distribution strategies. If, on the other hand, m is bigger than n then the complexity of any of these probability distance strategies is at least n^2 . For the purpose of plotting the graphs given in Section 3.1, we set m at 8562.5, since this was the average number of cells per grid across the 16 serial burglars' activity spaces. Below is an example of how we calculated the complexity for the linear probability distance strategy. The complexity analysis of the remaining four probability distance strategies is presented in Table I and the program used to calculate the complexity of the strategy is provided in Appendix A.

The *linear* strategy assumes that the likelihood of an offender living at a particular location declines by a constant amount with the distance from a crime site location. The probability of finding the home is highest near the crime location but drops off by a constant amount for each unit of distance until it falls to zero. The function f in this case is defined by

$$f(x) = \begin{cases} a + bx & \text{if } x \leq -a/b \\ 0 & \text{if } x > -a/b, \end{cases} \quad (8)$$

where a is typically set to some small positive value such as 10 and b is a constant that is given a negative sign (e.g., $b = -1$) to indicate that the likelihood of locating the offender's home declines with increasing distance from a crime. The cost of evaluating f in Eq. (8) is just two, since we require only one multiplication and one sum.

2.4. Predicting Home Locations

For all strategies, the area representing an offender's activity space was formed by arbitrarily expanding the minimum and maximum x and y crime location co-ordinates by 2000 m. This expansion ensured that the offender's crime locations would not be on the boundary of the total search space. Spatial analyses of the crime locations were computed using the statistical package *CrimeStat II* (Levine and Associates, 2000). An exception was the center of the circle strategy, which was applied manually using graphical instruments and checked by a second author. All 24 cases were tested from five to 10 crimes, with the crimes being added chronologically. In total, there were 1584 predictions made (strategies (11) \times offender homes (24) \times task complexity (6)).

2.5. Measuring the Accuracy of Geographic Profiling Strategies

As indicated in Section 1.2, there are various ways of measuring the accuracy of the predictions made by different geographic profiling strategies. Two common measures of accuracy are search cost (Canter *et al.*, 2000), or hit percentage (Rossmo, 2000), and error distance (Levine and Associates, 2000; Snook *et al.*, 2002, 2004). To reiterate, search cost and hit percentage are equivalent strategies that are measured as the percentage of cells, in an overlaid grid, that need to be searched (where the search is conducted in order of highest through lowest probability) to locate the cell that contains the offender's home (Canter *et al.*, 2000; Rossmo, 2000). However, these accuracy measures are restricted to probability distance strategies since spatial distribution strategies only provide one estimated home location. In contrast, because error distance refers to the crow-flight distance between the estimated home location and the actual home location, this measure can be used for both the spatial distribution strategies and the probability distance strategies. Consequently, it is desirable to use error distance to conduct comparisons across disparate strategies, and this method is used in the following analyses.⁷

3. RESULTS

3.1. Complexity Costs as a Function of Task Complexity

Figure 1 shows the relative magnitude of the upper bounds on the complexity costs of the 11 geographic profiling strategies obtained in Section 2, as a function of task complexity (i.e., the number of crimes in a series). The complexity costs are plotted on a logarithmic scale (base 10) because the costs associated with the strategies situated in the upper half of the graph were almost 2000 times that of those located in the lower part of the graph. As can be seen from Fig. 1, the spatial distribution and probability distance strategies differed considerably from each other in their level of complexity, with only relatively minor variations in complexity evident within the two groups. Moreover, the relative complexities among all strategies remained consistent from five through to 10 crimes, such that it is possible to rank each strategy according to its level of complexity. The ranking from low complexity to high complexity were: centroid (1),

⁷Crow-flight distances are typically used when studying criminal spatial activity in British or older North American cities (e.g., Boston) because it is not always possible to identify the likely route taken by the offender between his home and crime location. In any case, because we are addressing the relative (rather than absolute) performance of strategies, use of an alternative measure of distance (e.g., City-block) would yield equivalent results.

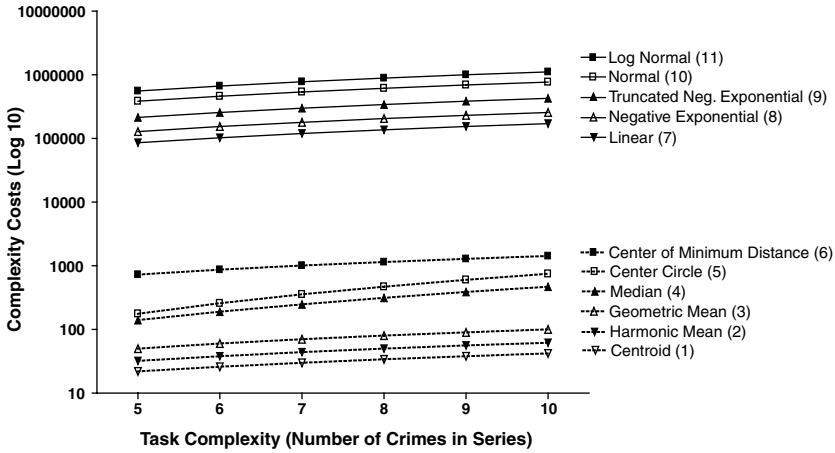


Fig. 1. Complexity costs for 11 geographic profiling strategies as a function of task complexity.

harmonic mean (2), geometric mean (3), median (4), center of the circle (5), center of minimum distance (6), linear (7), negative exponential (8), truncated negative exponential (9), normal (10), and lognormal (11).

3.2. Accuracy Measures by Strategy and Task Complexity

Table II contains the accuracy measures for each of the 11 geographic profiling strategies by task complexity. The strategies are presented in order from lowest complexity to highest complexity. The mean accuracy for each strategy is presented in the last column, and the mean accuracy for task complexity is presented in the last row. The bolded values refer to the most accurate geographic profiling strategy for a given level of task complexity.

As can be seen in Table II, there was no substantial difference in the accuracy of predictions across the strategies. Specifically, the mean accuracy across task complexity ranged from 8.01 km for the center of the circle to 9.06 km for the truncated negative exponential strategy. However, interestingly, the range between the most accurate and least accurate strategy does differ across task complexity. The center of the circle strategy produced the lowest mean error distance (i.e., 8.00 km) on the simplest task (i.e., five crime locations), which was 1.25 km more accurate than the most accurate probability distance strategy (i.e., linear) for that condition. In contrast, for the most complex task, the linear strategy was most accurate, yet, it was only 0.25 km more accurate than the least complex strategy (i.e., centroid), and only 0.06 km more accurate than the most accurate spatial distribution

Table II. Accuracy (km) for Each of the 11 Geographic Profiling Strategies by Task Complexity

Strategy	Task complexity: number of crimes in series						Mean accuracy
	5	6	7	8	9	10	
1. Centroid	8.39	8.14	7.98	8.07	7.96	7.91	8.07
2. Harmonic mean	8.40	8.14	7.98	8.08	7.96	7.91	8.08
3. Geometric mean	8.39	8.14	7.99	8.07	7.96	7.91	8.08
4. Median	9.25	8.51	8.56	8.44	8.15	7.84	8.46
5. Center of the circle	8.00	8.04	7.94	7.93	8.05	8.09	8.01
6. Center of minimum distance	8.99	8.44	8.46	8.42	7.93	7.72	8.33
7. Linear	9.25	8.71	8.60	8.46	8.17	7.66	8.47
8. Negative exponential	9.49	9.16	8.96	8.97	8.93	8.65	9.03
9. Truncated negative exponential	9.53	9.23	9.00	9.13	9.12	8.26	9.06
10. Normal	9.26	9.12	8.74	9.32	8.51	7.94	8.81
11. Log-normal	9.42	9.11	8.78	9.48	8.67	8.16	8.94
Mean accuracy	8.94	8.61	8.45	8.58	8.32	8.01	

strategy (i.e., center of minimum distance). Perhaps the most interesting finding from Table II, however, is the trend towards more complex strategies achieving the lowest mean error distance on tasks involving nine and 10 crimes. This finding suggests that complex probabilistic strategies may start to outperform simpler spatial distribution strategies on very complex tasks. To address this and other trends within the data, we turn to a statistical analysis.

A preliminary screening of the data indicated that accuracy distributions were leptokurtic (Range = 3.41–3.69), that is, there are too few accuracy scores in the tail of the distributions relative to the number of scores in the center of the distribution. Since the kurtosis requirement was violated, the data were not suitable for a parametric analysis. Although it was possible to substantially reduce this problem by removing three offenders from our sample, we conducted an initial analysis using all the data. We felt it might be important to include all the data on statistical grounds because the performance of strategies could differ dramatically on the offences that did not match the distribution. We also recognize the importance of including the data on conceptual grounds, since geographical profilers do not know in advance those cases that conform to the implicit assumptions of the geographical profiling strategies. With both of these arguments in mind, we felt that removing the crimes might provide a distorted view of strategy accuracy, and so chose to run two sets of analyses to provide a comprehensive understanding of the data.

The full data set was subjected to a two-way (strategies \times task complexity) Friedman's ANOVA (Marascuilo and McSweeney, 1977). There was no significant difference across task complexity when taking into account the type of strategy implemented ($F < 1$). Similarly, there was no significant difference across type of strategy while taking into account the number of crimes, $F_{(5, 776)} = 1.83$, *ns*. As a secondary analysis, we removed those maps responsible for the leptokurtic nature of the data (Maps 4, 9 and 17) and subjected the reduced data to a repeated-measures MANOVA. There was a significant main effect of task complexity, $F_{(5, 216)} = 8.49$, $p < 0.05$ (using Pillai's trace), but no significant main effect of strategy ($F < 1$) and no significant interaction between strategy and task complexity, $F_{(50, 1100)} = 1.08$, *ns*. Pair-wise comparisons were used to identify the differences in performance across task complexity, with Bonferroni adjustment made for multiple comparisons at the $\alpha < 0.05$ level. Comparisons revealed that the mean accuracy for five crimes was significantly worse than the accuracy for six or more crimes, that accuracy on nine crimes was significantly better than accuracy with eight crimes, and that the mean accuracy of strategies implemented on 10 crimes was significantly better (lower) than the accuracy across nine or less crimes. Overall, accuracy increased for all strategies with increasing task complexity.

4. DISCUSSION

Although geographic profilers have access to a repertoire of strategies for predicting a serial offender's home location, complex strategies are often prescribed over less complex strategies (Canter *et al.*, 2000; Rossmo, 2000). Indeed, computerized geographic profiling systems (and the probability distance strategies they employ) appear to be the generally accepted method for handling the geographic profiling task. One can find evidence for this by looking at the number of police agencies that have either purchased a geographic profiling system or have requested the assistance of geographic profilers who use such systems (Rossmo, 2000). This trend exists despite a lack of empirical evidence that more complex geographic profiling strategies lead to more accurate predictions than simpler strategies. Given the method we used to compute strategy complexity, this study showed that all probability distance strategies are substantially more complex than all spatial distribution strategies but are not more accurate. Our results also show that more complex strategies are not significantly more accurate than less complex strategies when the geographic profiling task is more complex (i.e., when the crime series of interest include more crimes). Instead, accuracy tended to increase with increasing task complexity across all strategies.

Although we must be cautious in drawing any firm conclusions from such a small-scale study, our results raise an important question: Is it necessary to use complex geographic profiling strategies to make accurate profiling predictions? Although the results presented here suggest not, we must qualify that statement with a number of other important points.

4.1. Our Definition of Complexity

In using our definition of complexity, we found substantial differences in the computational complexity of the various geographic profiling strategies (though these complexity measures are only approximations). For example, the simplest spatial distribution strategy (i.e., centroid) takes 22 steps to make a prediction when dealing with five crime site locations. In contrast, the simplest probability distance strategy (i.e., linear) takes 85,625 steps to make a prediction under the same condition. It is crucial to note that the complexity of a probability distance strategy is a function of the size of the number of cells in a grid that covers a criminal's activity space. If only a few cells are used to comprise the activity space (e.g., four or five cells, rather than 8000), then, the probability distance methods will be as simple as the spatial distribution methods. However, it is likely that using such a small number of cells would result in poor accuracy as the maximum center may be far from the actual home location (since there would be too few grid cell centers to choose from). Having said that, a prescribed advantage of geographic profiling systems, such as Dragnet (Canter *et al.*, 2000) and Rigel (Rossmo, 2000), is that they use grids that contain thousands of cells to increase precision. Consequently, we think that the number of grid cells included in our definition of complexity provide us with an appropriate measure of complexity.

It is also important not to automatically equate our definition of complexity with the amount of time and effort that would be expended when using these strategies in the real world. Once a complex probability distance strategy has been implemented within a computer program there is considerable scope for reducing the amount of time and effort that is needed to use the strategy. Thus, it is inappropriate to use evidence of differences in complexity but not performance as an argument for abandoning the use of probability distance strategies (and, therefore, geographic profiling systems) in favor of spatial distribution strategies.

4.2. Current Implementation of Complex Strategies

Although complex strategies do not necessarily require more time and effort to use in actual police inquiries, we believe there is an association

between complexity and effort. In large part, this association is the result of the way in which strategies are currently implemented. Probability distance strategies are usually incorporated into commercialized geographic profiling systems, which create financial, time and effort demands on the police because they must either purchase a system (and train one of their own officers to run it) or hire a geographic profiler. For example, if a police agency in North America decides to purchase a system, a senior officer who meets a number of criteria (e.g., at least 3 years experience investigating interpersonal crimes) must undergo a 2-year understudy training program (Rossmo, 2000), after which only he or she is able to conduct geographic profiles for the agency.⁸ Such implementation takes time, is costly, and leaves the agency in a position of depending on a particular officer. In contrast, spatial distribution strategies may be taught and later implemented within a very short time frame, as demonstrated by Snook *et al.*'s (2004) finding that performance improved after teaching a single sentence rule. This contrast is particularly important for police agencies that may require geographic profiles (e.g., due to high rates of serial crime in their jurisdiction) but do not have the resources to implement geographic profiling systems (e.g., due to limited time, money, technological capabilities, etc.). While geographic profiling services are available free of charge to law enforcement agencies in Canada, the US, and the UK, this is not the case in other countries. In these cases, low-cost, easy-to-implement geographic profiling strategies may be extremely useful, particularly if empirical evidence suggests that the simpler strategy can perform as accurately as a complex alternative.

4.3. Probability Distance Strategies go Beyond “X Marks-the-Spot”!

Another point that must be considered is whether spatial distribution strategies and probability distance strategies actually do the same thing. Recall that spatial distribution strategies provide only one estimate of the offender's likely residential location. Although probability distance strategies can also provide a single estimate of where the offender is most likely to live (i.e., the point of highest probability), they also provide a prioritized search strategy. Some geographic profilers argue that an ‘X Marks-the-Spot’ approach is a serious weakness of spatial distribution strategies and that it may be inappropriate to compare the probability distance strategies with spatial distribution strategies (Rossmo, 2000). Yet, while there is likely to be some truth in this argument, there is currently no evidence available to

⁸One of the *Journal's* anonymous referees drew our attention to the fact that a 2-week training course is now available to train police personnel on geographic profiling systems for property crimes (see <http://www.geographicprofiling.com/index.html>).

assess whether providing the police with a prioritized search area (or strategy) is useful.

This does not imply that geographic profiling systems are inaccurate, but rather questions the extent to which a prioritized search area will provide better accuracy than simply searching outward from the point of highest likelihood in a symmetrical manner. As other researchers have recognized, an answer to this question is likely to depend on the location of the offences. Indeed, Rossmo (2000) argues that police agencies must not only consider the hit rate of a method when considering the effectiveness of a geographic profile but also the size of the search area to determine what is feasible (e.g., it might be difficult for police in New York to effectively search a 2 km area that includes downtown Manhattan).

4.4. The Qualitative Component of Geographic Profiling

It has also been argued that geographic profiling involves a qualitative component that is based on the reconstruction and interpretation of the offender's mental map (Homant and Kennedy, 1998). Specifically, factors such as the hunting style of the offender, the density of potential victims, the location of major roads and highways, physical and psychological boundaries, and zoning and land use, are all taken into account to help refine a geographic profile once the quantitative prediction has been made (Rossmo, 2000). This approach is usually applied to geographic profiles derived from probability distance strategies, but there is no reason why this qualitative component could not be used to enhance profiles derived from other strategies. Indeed, a qualitative assessment of a quantitative profile is likely to be crucial regardless of the strategy that is used to make a prediction (e.g., at the very least a geographic profiler may want to make sure a prediction did not fall in an area that was completely uninhabitable, such as a lake). However, as noted by Levine and Associates (2000), it remains an open empirical question as to whether qualitative assessments can enhance the accuracy of quantitative geographic profiles.

4.5. Human vs. Actuarial Decision-Making

Previous research by Snook *et al.* (2002, 2004) has shown that people use simple strategies to make accurate geographic predictions. Some of these cognitive heuristics, such as the equidistant heuristic (choose a location that is equidistant from all crimes), tend to resemble some of the simpler strategies used in this research, such as the center of minimum distance (choose a location where the sum of the distance between that

location and all crimes sites is at a minimum). Consequently, future research should aim to explore whether particular geographic profiling strategies match the sorts of strategies that people use to predict where serial offenders reside. Assuming there is a match, the fact that simple strategies were shown to be as accurate as more complex strategies in this study further supports the idea that the simple heuristics people use can be as accurate as the complex strategies used in decision support systems.

5. CONCLUSION

This study showed that simple geographic profiling strategies are as accurate as complex strategies when predicting the likely home location of a serial offender based on information about where the offender committed their crimes. This was found to be the case regardless of the complexity of the geographic profiling task. Consequently, a geographic profiler may be able to maintain a comparable level of accuracy while minimizing their time and effort by using a relatively low-cost easy-to-implement geographic profiling strategy.

APPENDIX A. SPATIAL DISTRIBUTION STRATEGIES

Centroid

The centroid is a point whose coordinates are the mean of the x -co-ordinates and y -co-ordinates. The equation for deriving these coordinates is given in Table I, where x_i and y_i are the co-ordinates of crime locations and n is the total number of crime locations. Computing \bar{x} requires initially setting the variable SUM to zero, and then, for each n , adding the value of n to SUM and storing the new total, which on conclusion of the additions is divided by n . We count the retrieve-add-store operation as one step, which means that there will be always at most $2n$ steps (one for the setting of i to each value between 1 and n , and another one for each retrieve-add-store operation, plus one for the final output operation). The same program can be used to compute \bar{y} leading to: $T(n) \leq 2(2n + 1)$.

Median

If x is a real number then $\lfloor x \rfloor$ is the greatest integer less than or equal to x , and $\lceil x \rceil$ is the least integer greater than or equal to x (see for instance Cormen *et al.* (2001), page 51). The equation in Table I shows how the median is defined as the middle value of the distribution of the x -co-ordinates and

y -co-ordinates (David, 1970). A program for computing \bar{x} is achieved by initially sorting the coordinates and then, if n is even, setting \bar{x} to the value $\frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}$, while, if it is odd, setting \bar{x} to $x_{\frac{n+1}{2}}$. Letting $T_{\text{sort}}(n)$ be the time to sort n numbers, the total cost of this method is: $T(n) \leq 2(T_{\text{sort}}(n) + 4)$.

The expression for n even can be computed with two divisions (the one to compute $n/2$ and the *final* division by two) and two sums (the sum of one to $n/2$ and that of the two co-ordinates). We used a standard insertion sort algorithm to order the n numbers, whereby, given a list of n numbers one builds the final sorted list by repeatedly finding the smallest element in the given list and inserting it in the next available position of the output sequence (Cormen et al., 1990; Knuth, 1973). This algorithm, although of quadratic complexity in the worst case (it is fairly easy to implement insertion sort in time $2n^2 + 3n + 1$), performs well for the relatively low n explored in this study, and resulted in a step total of: $T(n) \leq 4n^2 + 6n + 10$.

Geometric Mean

The geometric mean is the anti-log of the mean of the logarithms. The equation for deriving of a set of n numbers is defined as the n th root of the product of the n numbers, but for our purposes the values of \bar{x} and \bar{y} are computed through the formulae contained in Table I. A program for computing \bar{x} involves setting the variable SUM to one, computing the natural logarithm of x_i , adding the result of the previous instruction to SUM, and storing the result in SUM. The value of SUM is then divided by n and the value of the anti-log of SUM (i.e., e^{SUM}) is computed. Although this program may look similar to the one used for the centroid, there are several important differences. Even if we charge one unit of time for each multiplication, the total cost of computing \bar{x} is $n \times (\text{LOG} + 2) + \text{EXP} + 1$, where LOG is the cost of computing the natural logarithm of a given number (assumed to be the same for all x_i 's), and EXP is the cost of computing the exponential function. This analysis leads to $T(n) \leq 2(n \times (\text{LOG} + 2) + \text{EXP} + 1)$ steps to compute \bar{x} and \bar{y} .

Harmonic Mean

The harmonic mean discounts extreme values of a distribution. As shown in Table I, in each dimension, it takes the inverse of the mean of the inverse of each coordinate. Computing \bar{x} initially requires setting the variable SUM to zero. Then, for each n , $1/x_i$ is computed and added to the current value of SUM, and is then stored in SUM. If the SUM is not zero then the output of the value of n is divided by SUM. Otherwise, the result is left undefined.

Since we charge one unit of time for each arithmetic operation, the total cost of computing \bar{x} , $3n + 1$ is bounded as follows: $T(n) \leq 2(3n + 1)$.

Center of Minimum Distance

The center of minimum distance of a given set of points in two-dimensional Euclidean space is the location at which the sum of the distances to all other points is the smallest. The equation for deriving these coordinates is given in Table I, where *dist* is the Euclidean distance between the point (x_i, y_i) and the chosen point (\bar{x}, \bar{y}) (Kuhn and Kuenne, 1962). Obtaining a precise value for the complexity of this strategy was more difficult than other spatial distribution strategies. The problem of minimizing *W* is not a trivial task, and it has a long history (see Drezner and Hamacher, 2002; Katz, 1974; Wesolowsky, 1993). The function, *W*, is convex so it has a unique minimum in the convex hull of the set of given points. As reported in Kuhn and Kuenne (1962), Weiszfeld (1936) developed an iterative algorithm that approximates the estimated home location through the iterative method:

$$(x^{(k+1)}, y^{(k+1)}) = \left(\frac{\sum_{i=1}^n x_i / \text{dist}((x_i, y_i), (x^{(k)}, y^{(k)}))}{\sum_{i=1}^n 1 / \text{dist}((x_i, y_i), (x^{(k)}, y^{(k)}))}, \frac{\sum_{i=1}^n y_i / \text{dist}((x_i, y_i), (x^{(k)}, y^{(k)}))}{\sum_{i=1}^n 1 / \text{dist}((x_i, y_i), (x^{(k)}, y^{(k)}))} \right)$$

where the initial value (x^0, y^0) may be taken as the centroid of the given set of points. Kuhn (1973) proved that the method actually converges to the center of minimum distance, but no precise estimate is known on how long it takes for the method to converge, as a function of the number of input points. The algorithm, which given $(x^{(k)}, y^{(k)})$, computes $(x^{(k+1)}, y^{(k+1)})$ involves computing the inverse of the distances between $(x^{(k)}, y^{(k)})$ and the *n* given points. Then, the sum of the *n* numbers computed in the previous step is computed, which is followed by the computation of $x^{(k+1)}$ and $y^{(k+1)}$.

It should be evident that each of these four steps runs in a number of time steps proportional to *n*. We employed this sequence of instructions in a program that computes an approximate value for the center of minimum distance (i.e., the value obtained by running Weiszfeld algorithm up to $k = 14$). The computational complexity of the resulting method can be bounded as follows: $T(n) \leq 14((9 + \text{ROOT}_2)n + 2)$.

PROBABILITY DISTANCE STRATEGIES

Negative Exponential

The negative exponential strategy assumes that the likelihood of an offender living at a particular location is highest near an offender's crime site location and decreases with increasing distance. As the formulae in Table I shows, the decline is at an exponential rate, dropping quickly near an

offender's crime location, as it approaches zero likelihood. This prediction is represented by one of a family of distance decay functions, where a is a coefficient (default = 10) used to provide an indication of the maximum likelihood of finding a home, and b is an exponent (default = 1) that determines the slope of the function. The cost of evaluating f is two plus the cost of evaluating the exponential function (EXP): $T_f = 2 + \text{EXP}$.

Normal

The normal strategy assumes the peak likelihood of an offender living at a particular location is at some optimal distance from the offender's crime site locations. The function rises to that distance and then declines. The rate of increase prior to the optimal distance, and the rate of decrease from that distance, is symmetrical in both directions. The mathematical form of this strategy is found in Table I, where the mean distance (μ) and the standard deviation (σ) of distances are fixed positive parameters, and a is a coefficient. A value of 1 is used as the default for μ , σ , and a .

For this strategy, where μ and σ are two positive real numbers, there are two steps for computing $\frac{x-\mu}{\sigma}$, one more step to square the result, and another one to divide it by -2 , equaling four steps. The result is multiplied by a , and divided by something that costs $2 + \text{SQRT}_2$. Therefore, the total cost is: $T_f = 7 + \text{EXP} + \text{SQRT}_2$.

Lognormal

The lognormal strategy is similar to the normal strategy except it is either positively or negatively skewed. This strategy predicts the offender should live near his crime sites and that there is a gradual decline in likelihood of locating the offender with increasing distance from his crime sites. The total cost of the lognormal strategy is:

$$T_f = 10 + \text{EXP} + \text{SQRT}_2 + \text{LOG}.$$

Truncated Negative Exponential

The truncated negative exponential strategy is a spline function consisting of the linear strategy and the negative exponential strategy. For points near the crime site locations, a positive version of the linear strategy (see Section 2.3.2) is used to represent the notion that offenders have a low likelihood of living directly on a crime location but an increased likelihood of living in the surrounding areas. At a predefined point of highest likelihood, the offender's "safe distance" away from the crime locations, the linear

strategy is replaced by the negative exponential strategy to represent a declining likelihood of locating the offender's home with increasing distance. The total cost of calculating the truncated negative exponential is the addition of the complexities associated with the two strategies.

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