MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

PHYSICS 4600 FINAL EXAMINATION – WINTER 2007 – APRIL 11, 2007

Name:________________________ **Student #:**____________________

INSTRUCTIONS:

1. Put your name and student number on each page.
2. **You must do Question #1 and select three questions from Questions #2 to #5.**
3. Question #1 is worth 40 marks (10 marks for each sub-question) and 20 marks for each of the other questions.
4. Constants and formulae are provided on the attached pages.
5. Use only the paper provided. No other books, notes or papers are permitted.
6. Do not remove examination papers from the examination room.

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1. 1a) (10 points) (i) Briefly describe the difference between thermal and photon detectors and their corresponding advantages. (ii) A laser beam is switched on at time $t = 0$, and is incident on a thermal power meter. If the detector response time is 15 s, at what time will the reading come to within 2% of the "true" value?

   1b) (10 points) Derive the ray matrix for a thin lens with a focal length of f.

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1c) (10 points) Explain the physical meaning of TEM_{mn} and give a representative sketch of irradiance or burn pattern for TEM_{21}.

1d) (10 points) Demonstrate that no second-order nonlinear optical phenomena occur in an isotropic optical material, or one having a center of symmetry.

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2. (20 points) A TEM_{00} He-Ne laser (λ = 632.8 nm) has a cavity that is 0.34 m long, a fully reflecting mirror of radius R = 10 m (concave inward), and an output mirror of radius R = 10 m (also concave inward).
   2a) From the symmetry of the mirror geometries and the boundary condition that wavefront and mirror curvatures match at the mirrors, determine the location of the beam waist in the cavity. Set z = 0 at this location to be the reference plane.
   2b) Determine the beam waist w_0.
   2c) Determine the beam spot size $w(z)$ at the left and right cavity mirrors.
   2d) Determine the half-angle beam divergence θ for this laser.
   2e) Where is the far field for this laser if you use the criterion $z_{FF} \geq 50(\pi w_0^2 / \lambda)$?
   2f) If the laser emits a constant beam of power 5 mW, what is the average irradiance at the position where $z_{FF} = 50(\pi w_0^2 / \lambda)$?

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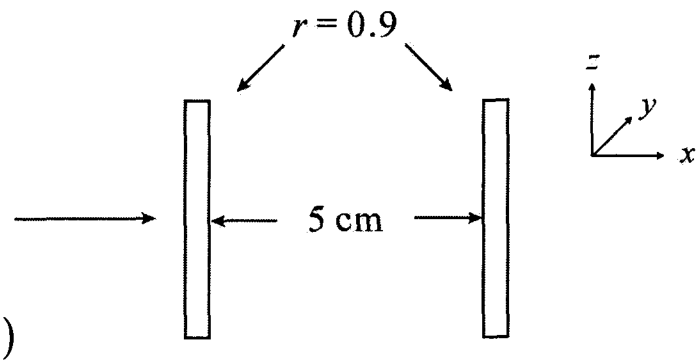
3. (20 points) Consider the following resonator, in which each mirror has a field reflection coefficient of $r = 0.9$ and the distance between the mirrors is 5 cm.



$$E = \hat{x}E_0 \exp(ikz - i\omega t)$$

If a field with amplitude E_0 is normally incident to the resonator and is polarized in the x direction, what is the total intensity inside the resonator? What is the mode spacing of the resonator and the full-width at half maximum of the resonance?

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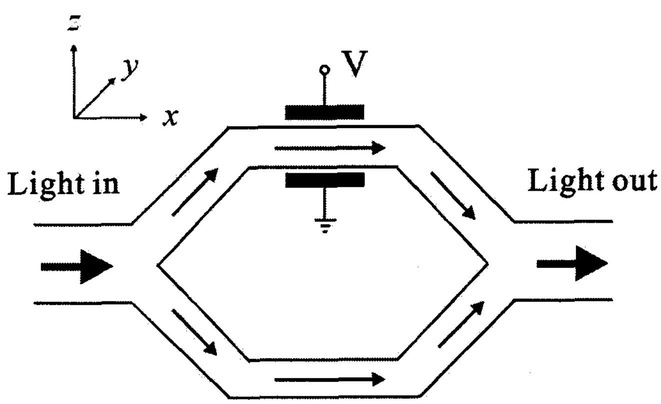
4. 4a) (10 points) An integrated-optical waveguide modulator uses $LiNbO_3$ (lithium niobate) in a Mach-Zehnder configuration. The crystal is orientated so that light propagates along the *x* axis, with the applied field and the light wave's field both along the *z* axis. The waveguide width (and spacing of electrodes) is 10 μm, and the length of the electrode region is 1 cm. Find the voltage V_π necessary to switch the output from low to high for light of free-space wavelength 1500 nm. The refractive index and the Pockels coefficient of $LiNbO_3$ at 1500 nm are 2.139 and 3.08×10^{-11} m/V, respectively.

   4b) (10 points) If we remove the voltage in 4a) and replace this region with a 1 cm-long gas cell with transverse dimensions 10×2 μm. Suppose maximum output is initially observed with light at wavelength 1500 nm of optical intensity 6×10^{13} W/m^2. When the cell is filled with an unknown third-order nonlinear optical material (χ_3 medium), 3 minima are observed before final maximum appears at the output. Calculate the nonlinear refractive index of the nonlinear optical medium. The influence of the windows of the gas cell is negligible.



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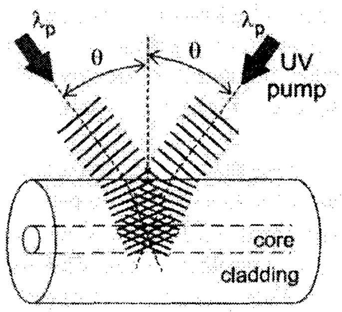
5. 5a) (6 points) Using the geometry in the following diagram, show that the Bragg grating spacing Λ in the fibre core is given by

$$\Lambda = \frac{\lambda_p}{2\sin\theta}$$

where λ_p is the wavelength of the pumping beams, and θ is the incident angle.



5b) (7 points) A fibre Bragg grating with λ_B = 1300 nm is written into the core of a silica fibre. The fibre used is highly doped with GeO_2, and has n = 1.5 and Δn = 2.5×10^{-4}. How far will light of wavelength 1300 nm propagate through the fiber grating before the light wave's E field is reduced to 1% of its initial value? At this point, how many grating periods has the light passed through?

5c) (7 points) Illustrate dispersion curves for two-dimensional photonic crystals and explain conditions when a complete photonic band gap occurs.

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CONSTANTS AND FORMULAE

Note: This list contains constants and formulae that may be or may not be useful for solving the questions in the examination.

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$C = 2.998\times 10^8 \text{ m/s}$$

$$E_{\text{photon}} = h\nu = \frac{hc}{\lambda}$$

$$\upsilon_p = \frac{\omega}{k} = \nu\lambda$$

$$\rho = \frac{1}{2}\varepsilon_r\varepsilon_0 E^2$$

$$I = \frac{c}{n}\rho = \frac{1}{2}cn\varepsilon_0 E^2$$

Optical detection:

$$mC\Delta\phi = (P_{\text{in}} - G\phi)\Delta t$$

$$\frac{d\phi}{dt} + \frac{G}{mC}\phi = \frac{P_{\text{in}}}{mC}$$

$$\phi(t) = \phi_{\max}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{mC}{G}$$

$$\phi_{\max} = \frac{P_{\text{in}}}{G}$$

$$i = \frac{P_{\text{in}}}{h\nu}e\eta$$

$$\Re \equiv \frac{i}{P_{\text{in}}} = \frac{e\eta}{h\nu}$$

$$G = \frac{Q}{e} = \delta^N$$

Optical resonator & laser dynamics:

$$U = \frac{1}{4}\varepsilon E_0^2 V$$

$$Q = \omega \times \frac{\text{field energy stored by resonator}}{\text{power dissipated by resonator}}$$

$$\frac{I_r}{I_i} = \frac{4R\sin^2(\delta/2)}{(1-R)^2 + 4R\sin^2(\delta/2)}$$
$$R \equiv r^2 = r'^2$$
$$\delta = \frac{4\pi nl\cos\theta}{\lambda}$$

$$\nu = m\frac{c}{2nL}$$

$$E_{l,m}(r) = E_0\frac{\omega_0}{\omega(z)}H_l\left(\sqrt{2}\frac{x}{\omega(z)}\right)H_m\left(\sqrt{2}\frac{y}{\omega(z)}\right)\times\exp\left(-\frac{x^2+y^2}{\omega^2(z)} - ik\frac{x^2+y^2}{2R(z)} - ikz + i(l+m+1)\eta\right)$$

$$\omega(z) = \omega_0\left[1+\left(\frac{z}{z_0}\right)^2\right]^{1/2},$$

$$z_0 = \frac{\pi\omega_0^2 n}{\lambda}$$

$$R(z) = z\left[1+\left(\frac{\pi\omega_0^2 n}{\lambda z}\right)^2\right] = \frac{1}{z}\left[z^2+z_0^2\right]$$

$$(\omega_0)_{\text{conf}} = \left(\frac{\lambda l}{2\pi n}\right)^{1/2},$$

$$\theta_{FF} \cong \frac{\lambda}{\pi\omega_0}$$

$$\eta = \tan^{-1}\left(\frac{\lambda z}{\pi\omega_0^2 n}\right)$$

$$0 \le \left(1-\frac{l}{R_1}\right)\left(1-\frac{l}{R_2}\right) \le 1$$

$$\tilde{q}_2 = \frac{A\tilde{q}_1 + B}{C\tilde{q}_1 + D}$$

Nonlinear optical phenomena:

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$$

$$P = \varepsilon_0 \left(\chi_1 E + \chi_2 E^2 + \chi_3 E^3 + \cdots \right)$$

$$\eta_{\text{SHG}} \equiv \frac{I^{(2\omega)}}{I^{(\omega)}} = \frac{2\omega^2 d^2 L^2}{n^3} \left(\frac{\mu_0}{\varepsilon_0} \right)^{3/2} \frac{\sin^2[(\Delta k)L/2]}{[(\Delta k)L/2]^2} I^{(\omega)}$$

$$L_c = \frac{\lambda_0}{4\Delta n}$$

$$\hbar \mathbf{k}_1 + \hbar \mathbf{k}_2 = \hbar \mathbf{k}_3$$

$$\hbar \omega_1 + \hbar \omega_2 = \hbar \omega_3$$

$$\frac{1}{n^2} = \frac{1}{n_0^2} + rE + RE^2$$

$$d\left(\frac{1}{n^2} \right) = rE$$

$$|\Delta n| \cong \frac{r}{2} n_0^3 E$$

$$\frac{1}{n^2} = \frac{1}{n_0^2} + RE^2$$

$$|\Delta n| = \frac{R}{2} n_0^3 E^2$$

$$\chi_1^{'} = \chi_1 + \frac{3}{4} \chi_3 A^2$$

$$n = \sqrt{\varepsilon_r} = \sqrt{\varepsilon / \varepsilon_0} = \sqrt{1 + \chi}$$

$$\Delta n = \frac{1}{2\sqrt{1 + \chi_1}} \Delta \chi_1 = \frac{\Delta \chi_1}{2n} = \frac{3}{8n} \chi_3 A^2$$

$$n \equiv n_0 + n_2 I$$

$$n_2 = \frac{3\chi_3}{4n^2 c \varepsilon_0}$$

Photonic crystal optics:

Step-index grating

$$r=\frac{E_r}{E_i}=\frac{n_1-n_2}{n_1+n_2}\cong-\frac{\Delta n}{2n}$$

$$\tilde{E}_{r,12}=-\frac{\Delta n}{2n}A\left[1+e^{-i\delta}+e^{-i2\delta}+\cdots+e^{-i(N-1)\delta}\right]$$

$$\delta=k(2\Lambda)=\frac{2\pi n}{\lambda_0}(2\Lambda)=\frac{4\pi n\Lambda}{\lambda_0}$$

$$\tilde{E}_{r,21}=\frac{\Delta n}{2n}Ae^{-i\delta/2}\left[1+e^{-i\delta}+e^{-i2\delta}+\cdots+e^{-i(N-1)\delta}\right]$$

$$\lambda_B=2n\Lambda$$

$$R_{\max}=\left|\frac{\tilde{E}_r}{\tilde{E}_i}\right|^2=\left|-\frac{\Delta n}{n}N\right|^2=N^2\left(\frac{\Delta n}{n}\right)^2$$

$R_{\max}=(\kappa L)^2$ (peak reflectivity, weak grating)

$\frac{\Delta\lambda}{\lambda_0}=\frac{1}{N}$ (spectral half-width, weak grating)

$$\kappa\equiv\frac{2\Delta n}{\lambda_B}$$

Sinusoidal index grating

$$n(x)=\bar{n}+\Delta n\cos\left(\frac{2\pi x}{\Lambda}\right)$$

$$\kappa\equiv\frac{\pi\Delta n}{\lambda_B}$$

$$R_{\max}=\tanh^2(\kappa L)$$

$$E(x)=E_0e^{-\kappa x}$$

$$\frac{\Delta\lambda}{\lambda_0}=\begin{cases}\lambda_0/(2nL) & \text{for } \kappa L \ll 1\\ \Delta n/(2n) & \text{for } \kappa L \gg 1\end{cases}$$