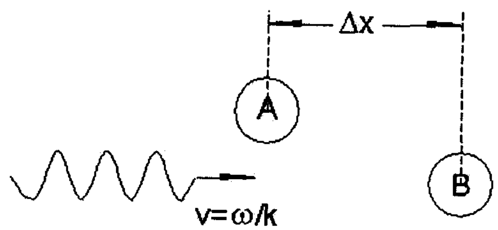
Physics 4600 — Final Examination (April 12, 2004)

Complete ANY FOUR questions

#1 **(a)** The process of light scattering by atoms and molecules can provide a plausible description of the manner in which light propagates through transparent media. As in the diagram at the right, consider a pair of immobile atoms (A & B) which act as scatterers to reemit light without change in frequency after a fixed time, Δt. Use a phasor diagram to show that light scattered in the forward direction will always interfere constructively while in the backward direction it generally will not. [7%]



(b) Likewise, assuming a spatially random distribution of scatterers in the light path, use a phasor diagram to show that the net **E** field amplitude in the backward direction should be zero. [6%]

(c) The effectiveness of the process in part **(a)** will depend on the density of scatterers. Can you suggest a qualitative dependence on density of the *net backward intensity per particle*? [6%]

(d) In real high-density, transparent materials (e.g. liquids) there is always low intensity, but observable, scattering of light in directions other than forward. What are the material properties that contribute to this effect ? [6%]

#2 **(a)** For a given uniaxial crystal the optic axis is along the z axis. Describe the behavior of the electric field (i.e. the polarization) of the following monochromatic light waves as they propagate through this crystal ($\vec{E}_o$ is the wave amplitude, a and b are arbitrary scalars). [11%]

$$(i)\quad \vec{k} = |\vec{k}|\hat{z}\ ;\ \vec{E}_o = a\hat{x} + b\hat{y}$$

$$(ii)\quad \vec{k} = |\vec{k}|\hat{x}\ ;\ \vec{E}_o = a\hat{z}$$

$$(iii)\quad \vec{k} = |\vec{k}|\hat{y}\ ;\ \vec{E}_o = a\hat{z}$$

$$(iv)\quad \vec{k} = |\vec{k}|\hat{x}\ ;\ \vec{E}_o = a\hat{y} + b\hat{z}$$

$$(v)\quad \vec{k} = |\vec{k}|\hat{y}\ ;\ \vec{E}_o = a\hat{x} + b\hat{z}$$

$$(vi)\quad \vec{k} = |\vec{k}|\hat{y}\ ;\ \vec{E}_o = a(\hat{x} + \hat{z})$$

(b) For the case of $\vec{k} = |\vec{k}|\hat{x}$ sketch a typical contour of constant refractive index for all possible directions of the electric field (perpendicular to $\vec{k}$). [7%]

(c) Describe how such a material could be used to convert (incident) linear polarization to (exit) circular polarization. [7%]

#3 **(a)** According to the Drude Model the motion of electrons in a metal is subject to damping forces but otherwise the motion is not influenced by the parent ions. Write down, and identify the individual terms in, the differential equation of motion for these electrons in the presence of a harmonic driving force. [5%]

(b) Make a sketch of the refractive index of a typical metal as a function of frequency. [4%]

(c) Write down, and identify the individual terms in, the differential equation of motion for harmonically driven electrons in a dielectric. Describe how you would proceed to solve this equation to find an expression for the (real) refractive index as a function of frequency, i.e. $n(\omega)$. [A formal solution is not expected.] [8%]

(d) What is the basic functional form of $n(\omega)$ from part **(c)** ? Illustrate with a sketch. [4%]

(d) Make a qualitative sketch of $n(0<\omega<\infty)$ for a typical optical material such as SiO_2 and identify any distinctive features you would expect to be present due to electrons only (i.e. ignore possible phonon contributions). [4%]

#4 **(a)** Given that f(t) and $F(\omega)$ are a Fourier Transform pair, derive Parseval's Formula : [12%]

$$\int_{-\infty}^{+\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |F(\omega)|^2 \, d\omega.$$

(b) Define the autocorrelation of f(t) and the cross-correlation of functions g(t) and h(t). [6%]

(c) What is the Wiener-Khintchine theorem and why is it significant with respect to spectral analysis ? [7%]

#5 **(a)** Make a sketch of the general form of the dispersion curve(s) for phonons in a crystal and identify the important features. [5%]

(b) What are the essential requirements for phonons to cause absorption of light, what types of phonons are primarily involved, and in what region of the EM spectrum does the absorption typically occur ? [5%]

(c) Use sketches to show how phonons affect the reflection spectrum of a typical crystal ? In particular, what is *restrahlen* ? [5%]

(d) What are *polaritons* and how do they modify the dispersion curves of part **(a)** ? [5%]

(e) What are *polarons* ? How do they affect the effective mass of electrons and the band gap in semiconductors ? [5%]

End