Physics 4300. Final exam

Saturday, April 16, 2005, Time: 2 hours
Closed book exam.

Problem 1.

i) It is well known that the fragments of tea leaves at the bottom of a stirred cup of tea conglomerate towards the center. Explain this phenomenon with Ekman-layer dynamics. Also explain why the tea leaves go to the center irrespective of the direction of the stirring (clockwise or counterclockwise).

ii) Assume that immediately after the stirring the water in the cup of radius R = 5 cm rotates as a solid body with velocity U = 5 cm/s at the side wall of the cup (neglect a boundary layer at the side wall). Estimate the net transport of water towards the center of the cup (in the radial direction) in the boundary layer at the bottom of the cup. What is the average radial velocity of the tea leaves in the boundary layer? Take the value of the kinematic viscosity to be $\nu = 0.01\ \mathrm{cm^2/s}$.

Problem 2.

i) Derive the equation of conservation of potential vorticity;

$$\frac{d}{dt}\left(\frac{f+\zeta}{H+\eta}\right)=0 \qquad (1)$$

starting with the equation of absolute vorticity conservation:

$$\frac{d}{dt}(f+\zeta)+(f+\zeta)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \qquad (2)$$

and the equation of continuity for shallow water and a flat bottom,

$$\frac{d\eta}{dt}+(H+\eta)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \qquad (3)$$

ii) Explain (making reference to equation 2) how the relative vorticity of a column of water would change due to convergence.

iii) Consider the situation where the depth decreases in the direction of flow. In the northern hemisphere, how would you expect the relative vorticity of a parcel of water to change as it moves:

From east to west?
From north to south?
From south to north?

Problem 3.

Tidal motions behave as waves in the open ocean. For these long period waves, the earths rotation is important and these waves are governed by the equations of motion for *gravitational - gyroscopic* waves. For open ocean waves, the Poincare wave is a suitable model and these waves have the dispersion relation:

$$f^2 - \omega^2 + gk^2H = 0.$$

Here, f is the Coriolis parameter, ω is the angular frequency of the wave, g is the acceleration due to gravity, k is the wavenumber, and H is the water depth.

Find the phase velocity for a semi-diurnal tide in the deep ocean at 45^0 N latitude. Compare this speed to the speed at which the earth's surface moves underneath the sun at this latitude (ie. how fast would you have to travel along a line of latitude to keep the sun directly above you?). What consequences would this difference imply?

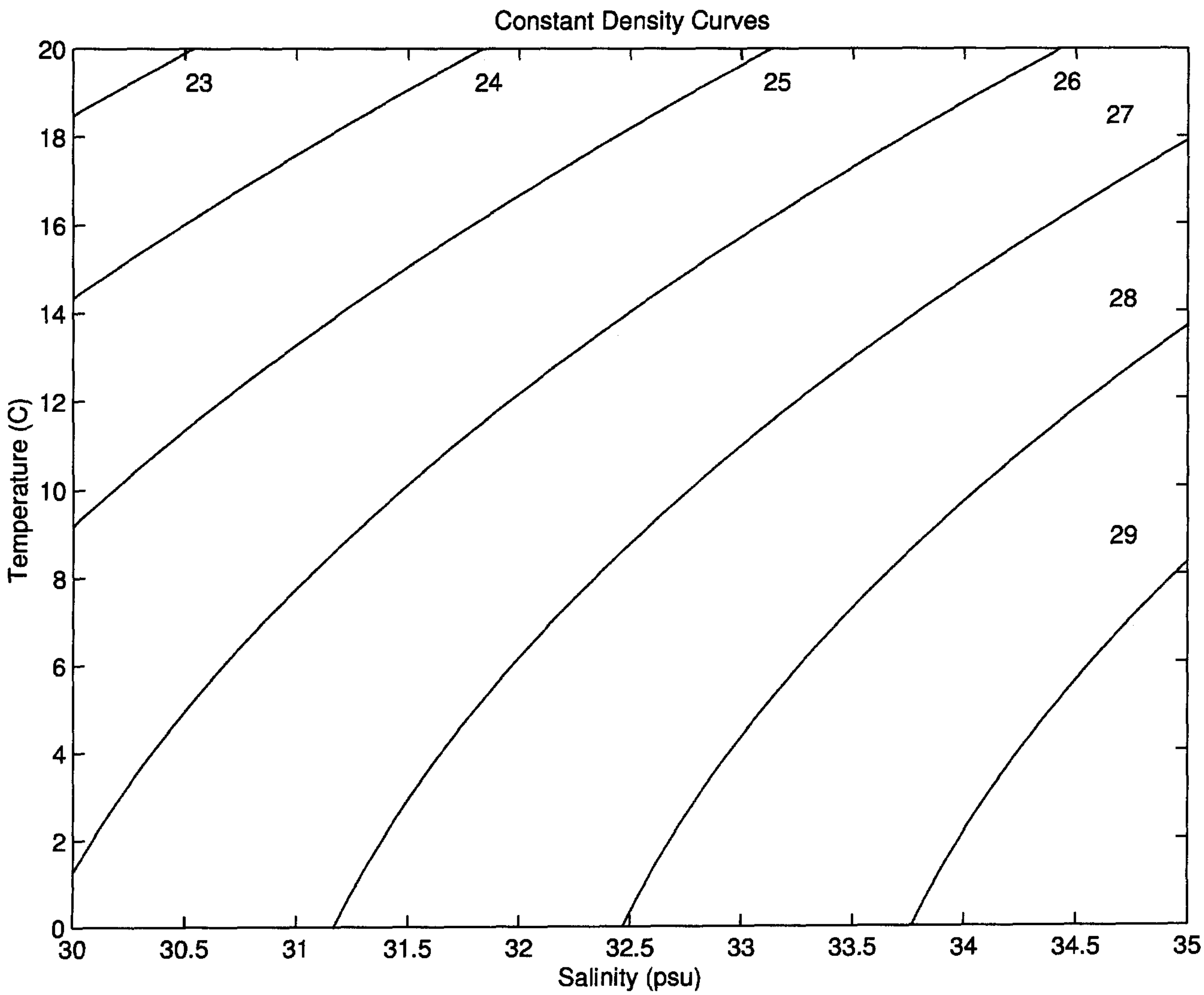
Problem 4.

Water types A, B, C and D have the following properties:

Water Type A: Temperature = 12.3 C, Salinity = 30.45 psu
Water Type B: Temperature = 12.3 C, Salinity = 34.30 psu
Water Type C: Temperature = 2.50 C, Salinity = 32.00 psu
Water Type D: Temperature = 4.95 C, Salinity = 32.58 psu

i) Using the Figure attached determine graphically what combination and volume of water types A, B, and C must be mixed together in order to arrive at the properties of water type D. Obtain the same result algebraically.

ii) How would water types A, B, C and D be distributed vertically relative to each other in a stably stratified ocean?


Constant Density Curves
Temperature (C)
Salinity (psu)
23
24
25
26
27
28
29
0
2
4
6
8
10
12
14
16
18
20
30
30.5
31
31.5
32
32.5
33
33.5
34
34.5
35