Phys3751 Final Exam

Physics Department
Course: Phys 3751 Final Exam
Date: Monday, April 9, 2007.
Time 9:00 – 11:00 AM
Instructor Anand Yethiraj.

ANSWER ALL QUESTIONS: Each question is worth 20 points.
Show all steps in your reasoning and you will get partial credit.

(I) PARTICLE PHYSICS PHENOMENA AND FEYNMAN DIAGRAMS

(A) A significant part of experimental particle physics involves obtaining information from charged particle tracks.

(i) If a particle with charge q and mass m) is undeflected in passing through uniform crossed electric and magnetic fields E and B (mutually perpendicular and both perpendicular to the direction of motion), what is the particle's velocity v in terms of q , E and B ?

(ii) If we now turn off the electric field, and the particle moves in an arc of radius R , what is its charge-mass ratio in terms of R , E and B ?

(B) Sketch all the distinct lowest order Feynman diagrams representing Delbruck scattering:
$\gamma+\gamma\rightarrow\gamma+\gamma$.

(C)

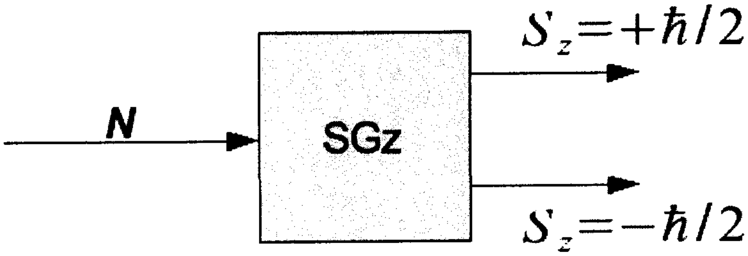
(i) Draw Feynman diagrams for the two processes:
$\Sigma^- \rightarrow \Lambda+\pi^-$ and $\Sigma^- \rightarrow n+\pi^-$

(ii) Which decay do you think is more likely and why?

(II) SPIN AND THE STERN-GERLACH EXPERIMENTS

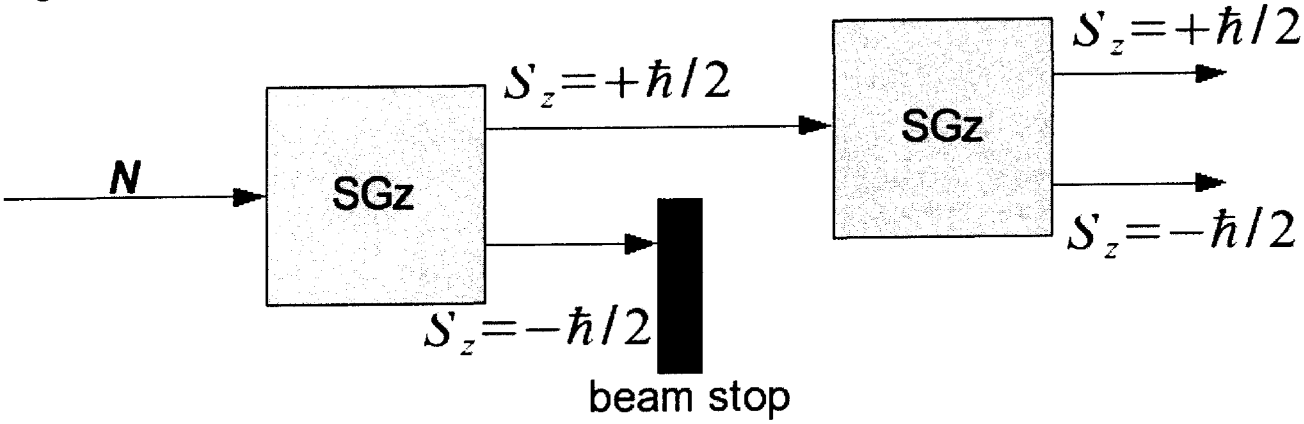
(A) The diagram below shows an SGz device (a Stern-Gerlach device whose inhomogeneous magnetic field points in the z direction) that splits a beam of *N* spin-1/2 particles. Into two beams one with $S_z=+\hbar/2$ and the other where $S_z=-\hbar/2$. What is the expected number of particles in each of these two states ?

Diagram 1



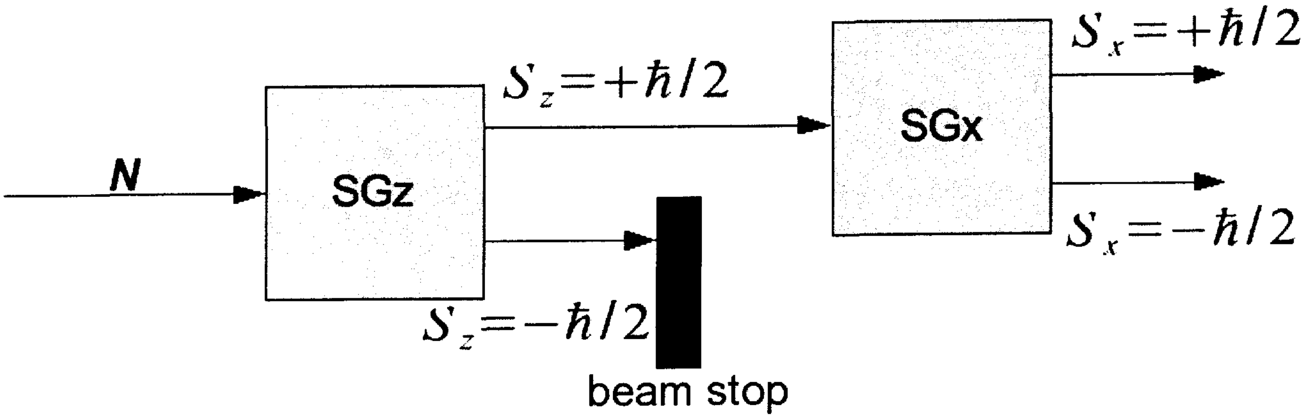
(B) Diagram 2 below shows an extension of Diagram 1. Here the beam with $S_z = +\hbar/2$ is allowed to pass through to **a second SGz device**. This second device produces two beams with $S_z = +\hbar/2$ and $S_z = -\hbar/2$. What is the expected number of particles in each of these two states ?

Diagram 2



(C) Diagram 3 shows a different extension of Diagram 1. Here the beam with $S_z = +\hbar/2$ is allowed to pass through to **a second SGx device**. This second device produces two beams with $S_x = +\hbar/2$ and $S_x = -\hbar/2$. What is the expected number of particles in each of these two states ?

Diagram 3



(D) A particle is in an arbitrary quantum state $|\psi\rangle = c_+ |+z\rangle + c_- |-z\rangle$. An S_z measurement is carried out on this state (for example, by a Stern-Gerlach SGz experiment)

(i) What is the **amplitude** of a measurement of S_z on this state yielding $S_z = +\hbar/2$?

(ii) What is the corresponding **probability**?

(iii) Use the normalization of probability to obtain an equation that relates the coefficients c_+ and c_- .

(III) SYMMETRIES AND SPECIAL RELATIVITY

(A) Emmy Noether's theorem is a very important result in theoretical physics that relates continuous symmetries to conservation laws. Which conservation laws do each of the following symmetries correspond to:

(i) (Time) translation symmetry

(ii) (Space) translational symmetry

(iii) (Space) rotational symmetry

(iv) Gauge symmetry in Maxwell's equations

(B) Particle A (mass m_A , at rest) decays into particle B (mass m_B) and particle C (mass m_C). Obtain a relation for the total energy of [redacted] the outgoing particle B in terms of the various masses and the speed of light.

(IV) ABC THEORY

(A) Consider the general case of two-body decay $A \rightarrow B + C$, where all particles have mass. To lowest order, the amplitude for the decay of the A:

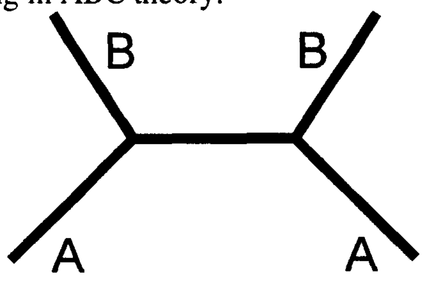
$\mathcal{M} = g$, where g is the coupling constant in ABC theory (and a real number).

Apply the general equation for the differential decay rate (provided below) to this problem, and obtain a formula for the decay rate. You do not need to evaluate the value of the outgoing momentum that is obtained after integration in terms of the masses – simply call it ρ_0 .

Fermi Golden Rule for decays:

$$d\Gamma = |\mathcal{M}|^2 \frac{S}{2\hbar m_1} \left(\frac{c\, d^3 \vec{p}_2}{(2\pi)^3 2E_2} \right) \left(\frac{c\, d^3 \vec{p}_3}{(2\pi)^3 2E_3} \right) \cdots \left(\frac{c\, d^3 \vec{p}_n}{(2\pi)^3 2E_n} \right) \times (2\pi)^4 \delta^4 (p_1 - p_2 - p_3 - \dots p_n)$$

(B) Scattering in ABC theory.



(i) Consider the following Feynman diagram (representing the scattering of 2 A particles) in ABC theory : $A+A\rightarrow B+B$.

(ii) What is the identity of the unlabeled line (i.e. what particle is it)?

(iii) Use the Feynman rules provided below to obtain the lowest order contribution to the amplitude $\mathcal{M}$ for this diagram.

Feynman Rules for ABC Theory

1. Notation: Label the incoming and outgoing momenta with p indices and internal vertices with q indices and with arrows to keep track of signs in (4).
2. For each vertex write down a factor of $-ig$.
3. Propogator. For each internal line, write a factor
$$\frac{i}{q_j^2-m_j^2c^2}$$
4. Conservation of energy and momentum. For each vertex write down a delta function of the form $(2\pi)^4\delta^4(k_1+k_2+k_3)$
5. For each internal line write down a factor $\frac{1}{(2\pi)^4}d^4q_j$ and integrate over all internal momenta.
6. Cancel the delta function factor $(2\pi)^4\delta^4(p_1+p_2...-p_n)$. What remains is $-i\mathcal{M}$.