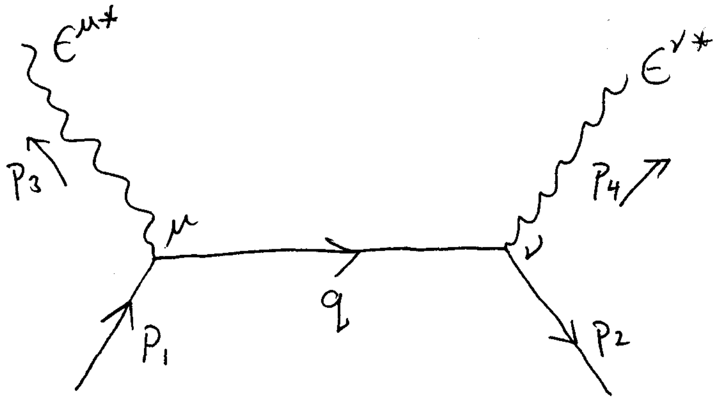
PHYS3751 Quantum Physics II Final Examination
Thursday, April 14, 2005, 9-11am

Instructor: S. H. Curnoe

Instructions: Complete all four questions. Each question is worth 25%. You have two hours.

1. **Quantum Electrodynamics (QED)**

   Consider the diagram:



   (a) Use the Feynman rules (see attachments) to write down the amplitude $\mathcal{M}$ for this diagram. (Do the integration over q.)

   (b) This diagram is $O(e^2)$ (second order in e). Draw another diagram which for the same process which is also $O(e^2)$.

   (c) There are 16 diagrams which are $O(e^4)$. Draw six of them.

2. **Nuclear Physics**

   (a) Consider $^3Li^7$ and $^4Be^7$, with masses 7.010600 a.m.u and 7.0169 a.m.u. respectively.

      i. Which one can decay into the other?

      ii. What kind of decay will it be?

      iii. How much energy will be released, and where does it go?

   (b) Use the shell model to find the possible values of the nuclear spins and the parities of: $^5B^{10}$ and $^{19}K^{40}$.

   (c) Of the four nuclear models we discussed in class (Shell, Fermi Gas, Liquid Drop and Collective) which one gives the best explanation for the fact that the number of protons is approximately equal to the number of neutrons in a nucleus? Justify your answer.

   (d) Discuss similarities and differences between the problem of finding energy levels of an atom versus the energy levels of a nucleus.

3. **Superconductivity**

   (a) Describe

      i. the penetration depth and

      ii. the coherence length.

   (b) What is the "intermediate state" of Type I superconductors?

   (c) Given

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

   and

$$\vec{M} = \chi\vec{H},$$

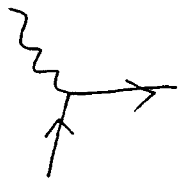
   where $\vec{H}$ is an applied magnetic field, $\vec{B}$ is the total internal magnetic field and $\vec{M}$ is the magnetisation, what is χ for a superconductor?

   (d) When a small superconducting sample is placed on top of a magnet it *levitates* (rises above the magnet). Explain.

4. **Miscellaneous**

(a) What is the Fermi Sea in the context of particle physics? How much energy does it cost to excite an electron with energy $-E$, momentum $\vec{p}$ and spin m_s, to energy E? What are the energy, momentum, mass and spin of the hole?

(b) Consider the bare QED vertex:



This vertex is effectively *renormalised* by higher order diagrams. Draw three diagrams which contribute to the renormalisation of the vertex.

(c) In general, spin-orbit coupling lifts the degeneracy of a $(2s+1)(2l+1)$-fold degenerate level. Consider $s = 1/2$ and $l = 3$. What are the possible values of the total angular momentum j and the degeneracies of the new levels?

(d) The Mossbauer effect is based on "recoilless emission" of a photon. How can you have recoilless emission and still conserve momentum?

Useful Stuff

$m_e = 0.511\text{MeV/c}^2$

$1 \text{ a.m.u} = 931.5 \text{ MeV}/c^2$

1. *Notatic*   abel the incoming and outgoing four-momenta $p_1, p_2, \ldots, p_n$, and the corresponding spins $s_1, s_2, \ldots, s_n$; label the internal four-momenta $q_1, q_2, \ldots$. Assign arrows to the lines as follows: the arrows on *external* fermion lines indicate whether it is an electron or a positron; arrows on *internal* fermion lines are assigned so that the "direction of the flow" through the diagram is preserved (i.e., every vertex must have one arrow entering and one arrow leaving). The arrows on external photon lines point "forward"; for internal photon lines the choice is arbitrary. (See Fig. 7.1.)

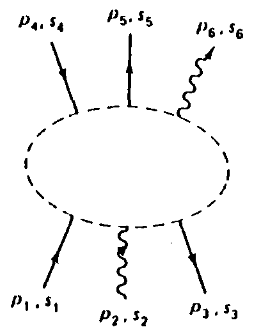


Figure 7.1 A typical QED diagram, with external lines labeled. (Internal lines not shown.)

2. *External Lines.* External lines contribute factors as follows:

Electrons:	Incoming (↗): u	Outgoing (↗): $\bar{u}$
Positrons:	Incoming (↗): $\bar{v}$	Outgoing (↗): v
Photons:	Incoming (~): ϵ^μ	Outgoing (~): $\epsilon^{\mu *}$

3. *Vertex Factors.* Each vertex contributes a factor

$$ig_e\gamma^\mu$$

The dimensionless coupling constant g_e is related to the charge of the positron: $g_e = e\sqrt{4\pi/\hbar c} = \sqrt{4\pi\alpha}$.*

4. *Propagators.* Each internal line contributes a factor as follows:

Electrons and positrons: $$\frac{i(\gamma^\mu q_\mu + mc)}{q^2 - m^2c^2}$$

Photons: $$\frac{-ig_{\mu\nu}}{q^2}$$

5. *Conservation of Energy and Momentum.* For each vertex, write a delta function of the form

$$(2\pi)^4\,\delta^4(k_1 + k_2 + k_3)$$

where the k's are the three four-momenta coming *into* the vertex (if an arrow leads *outward*, then k is *minus* the four-momentum of that line, except for external positrons*). This factor enforces conservation of energy and momentum at the vertex.

6. *Integrate Over Internal Momenta.* For each internal momentum q, write a factor

$$\frac{d^4q}{(2\pi)^4}$$

and integrate.

7. *Cancel the Delta Function.* The result will include a factor

$$(2\pi)^4\delta^4(p_1 + p_2 + \cdots - p_n)$$

corresponding to overall energy-momentum conservation. Cancel this factor, and what remains is $-i\mathcal{M}$.

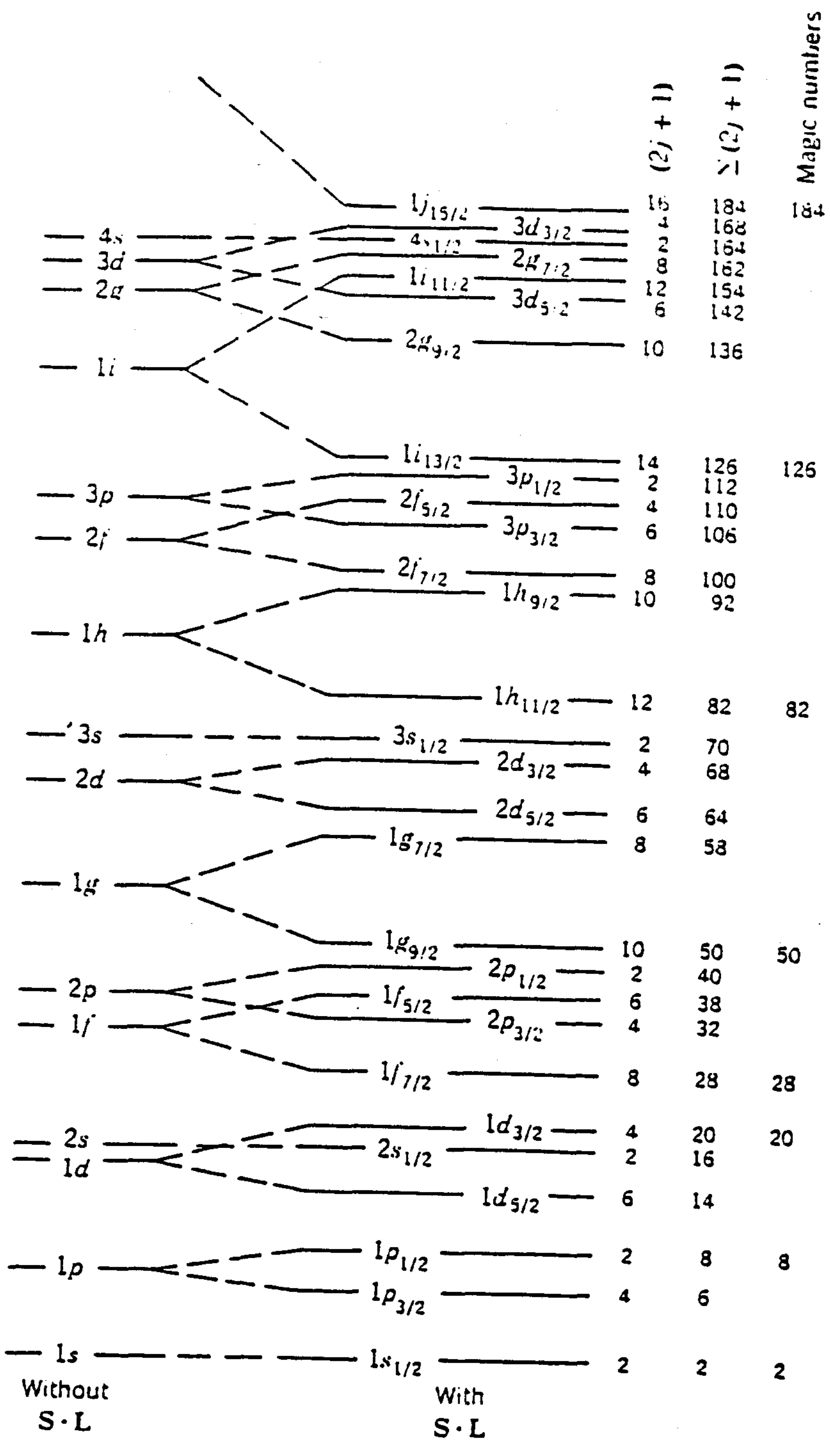


Figure 15-18 *Left:* The order of filling, as the occupancy and well radius increase, of the levels of rounded edge square wells with no spin-orbit interaction. *Right:* The levels that arise when a strong inverted **S · L** interaction is added. The column marked (2*j* + 1) shows the number of like nucleons that may occupy the corresponding level without violating the exclusion principle. The column marked Σ (2*j* + 1) gives for each level the cumulative number of nucleons that lie in all levels up through that level. Significant energy gaps lie above each of the levels marked with a magic number in the last column.