Engineering 3821/Physics 3550
Final Exam
December 14, 2002
Time: 2½ hours

Name:____

Student number:________________________________

Tick one:

Engieering 3821 ☐ **Physics 3550** ☐

This exam consists of eight (8) pages in total. Answer all six (6) questions. All questions have equal value.

You are permitted one (1) two-sided page of notes. No other notes, books, etc. are allowed.

It is assumed that students are familiar with the rules governing dishonest behaviour on examinations. If you are not familiar with these rules please ask for clarification. In particular, copying from another student, looking at another student's exam paper, or exposing your paper to another student are not permitted. Students found guilty of such behaviour will be penalized to the full extent permitted by university regulations.

Tables of Laplace transforms are given on the next page.

TABLE 12.1

An Abbreviated List of Laplace Transform Pairs

TYPE	$f(t)\ (t > 0^-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

TABLE 12.2

An Abbreviated List of Operational Transforms

OPERATION	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

TABLE 12.3

Four Useful Transform Pairs

PAIR NUMBER	NATURE OF ROOTS	$F(S)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2} + \frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $\lvert K\rvert\angle\theta$.

1) For the circuit shown,

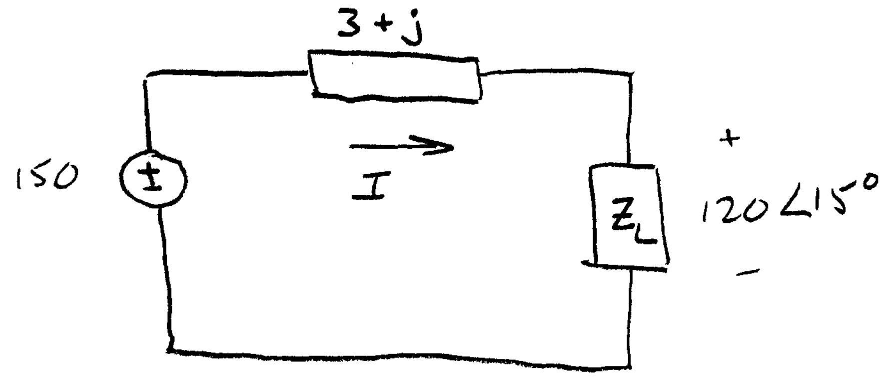
a) find the phasor current I

b) find the real power P, reactive power Q, and complex power S associated with the load impedance Z_L

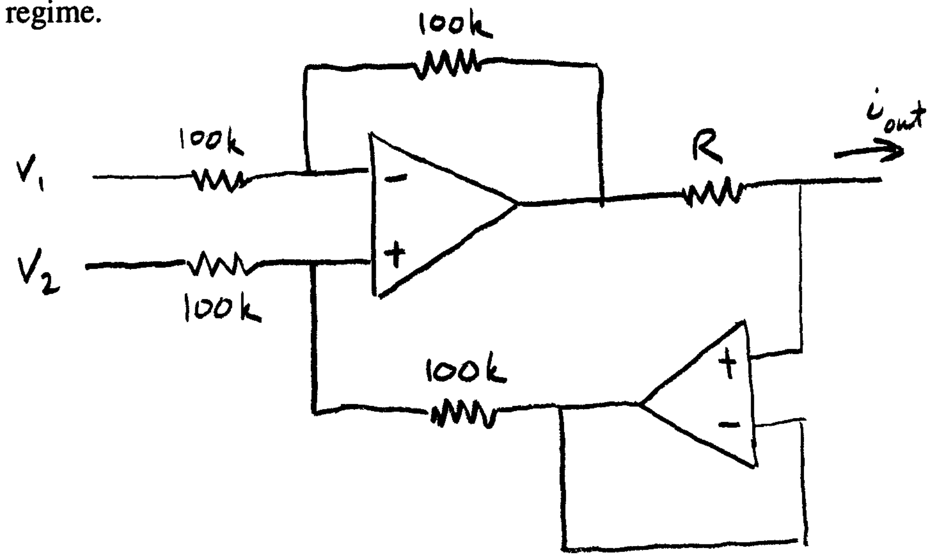
c) find the power factor angle for the load, θ.

d) draw a diagram showing the relationship between S, θ, the average power P and the reactive power Q.

The voltages given are all RMS voltages. Don't forget units as required.



2) Show that $i_{out} = (V_2 - V_1)/R$. Assume that the op-amps are operating in the linear regime.

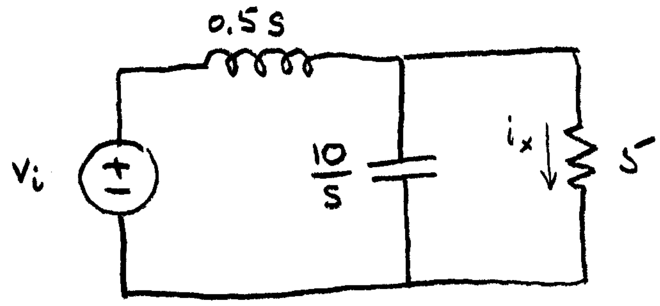


3)

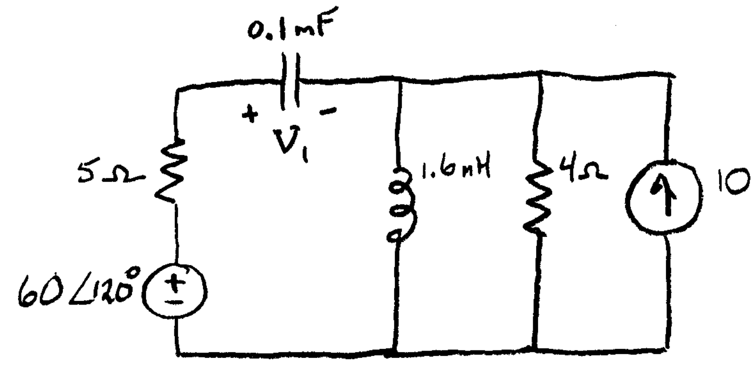
a) Find the transfer function *H(s)* for the circuit shown, where v_i is the input and i_x is the output.

b) Find $H(j\omega)$ for $\omega = 10$ rad/s.

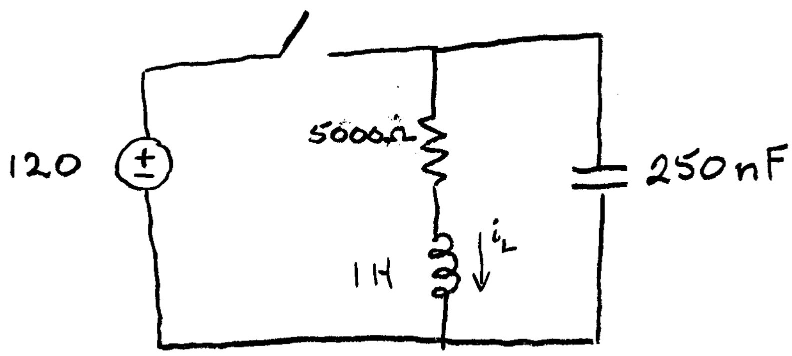
c) Find the steady state output of this circuit for $v_i = 17\cos\left(10t + 40^\circ\right)$



4) In the circuit shown, $\omega = 5000$ rad/s. Using phasors find the phasor voltage V_1 and the voltage $v_1(t)$.



5) At $t = 0$ the switch is suddenly closed. There is no energy stored in the circuit for $t < 0$. Using the technique of your choice, find i_L for $t > 0$.



6)

a) Design and draw a band pass filter based on a series LCR circuit having center frequency 1250 Hz and bandwidth 75 Hz. Use a 10 mH inductor. Don't forget to convert frequencies from Hz to radians/sec.

b) What is Q for your circuit?

c) What is the transfer function $H(s)$ for your circuit?

d) Without calculating the K values, describe the transient response of your circuit.