MEMORIAL UNIVERSITY OF NEWFOUNDLAND

Department of Physics and Physical Oceanography

Physics 3230 (Classical Mechanics II)

**Final Examination**

**April 30 , 2006, 3:00-5:00 p.m.**

Answer all 5 questions

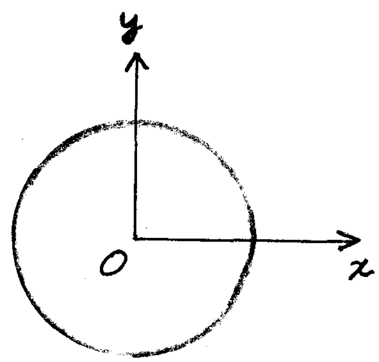
**Formulae**

$$
\begin{aligned}
r &= \frac{c}{1+\varepsilon\cos\phi} \\
c &= \frac{l^2}{\gamma\mu} \\
\gamma &= GmM \quad \mu = \frac{mM}{m+M} \\
g &= \frac{GM}{R^2}
\end{aligned}
$$

$$[\widetilde{K}] = [M]^{-\frac{1}{2}}[K][M]^{-\frac{1}{2}} \qquad [S] = [M]^{-\frac{1}{2}}[P] \quad \{r\} = [S]^{-1}\{x\}$$

$$
\begin{aligned}
\mathbf{v} &= \mathbf{v}_{O'} + \mathbf{v}' + \boldsymbol{\omega}\times\mathbf{r}' \\
\mathbf{a} &= \mathbf{a}_{O'} + \mathbf{a}' + 2\boldsymbol{\omega}\times\mathbf{v}' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}') + \boldsymbol{\alpha}\times\mathbf{r}'
\end{aligned}
$$

$$
\begin{aligned}
\{H\} &= [I]\{\omega\} \\
p_k &= \frac{\partial L}{\partial \dot{q}_k} \\
H &= \sum_{i=1}^{n} p_i\dot{q}_i - L \\
\dot{q}_k &= \frac{\partial H}{\partial p_k} \quad (k=1,\ldots n) \\
\dot{p}_k &= -\frac{\partial H}{\partial q_k} \quad (k=1,\ldots,n)
\end{aligned}
$$

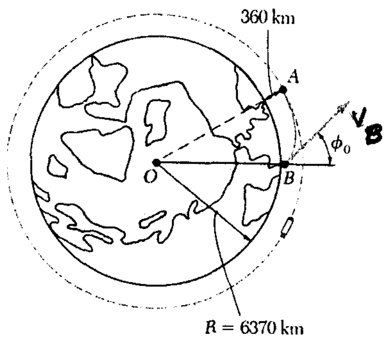



Disk of mass $m$, radius $r$

$$I_{xx} = I_{yy} = \frac{1}{4}mr^2$$

$$I_{zz} = \frac{1}{2}mr^2$$

1. A space shuttle is to rendezvous with an orbiting laboratory $A$ which circles the earth (centre $O$, mass $M$, radius $R$) at a constant altitude of 360 $km$. The shuttle $B$ has reached an altitude of 60 $km$ when its engine is shut off, and its velocity $\vec{v}_B$ forms an angle $\phi_0 = 50^0$ with the vertical $OB$ at that time. $R = 6370$ $km$.



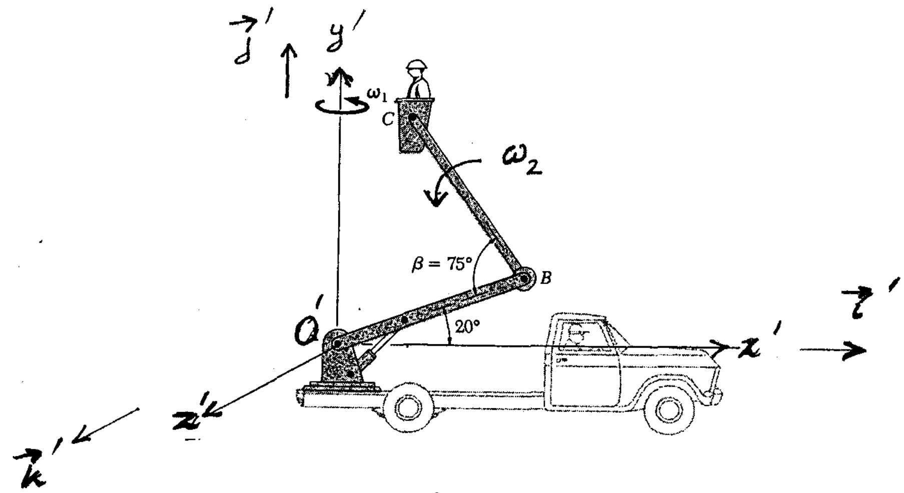

   (a) **[5 marks]** Using conservation of energy, show that

$$v_A^2 = v_B^2 - \frac{2GM}{r_B}\left(1 - \frac{r_B}{r_A}\right)$$

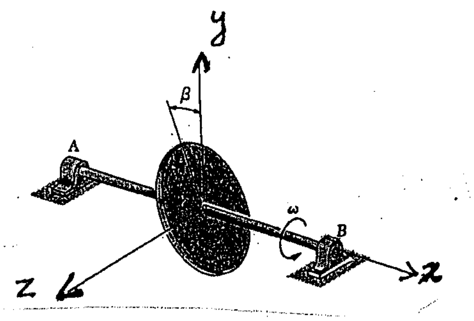
   where $r_A = OA$, $r_B = OB$ and the speeds at $B, A$ are $v_B, v_A$ respectively. $G$ is the gravitational constant.

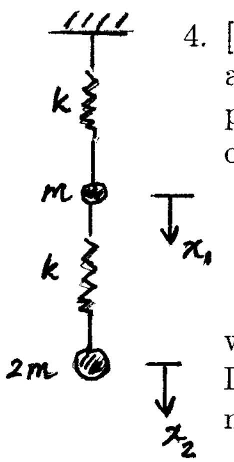
   (b) **[15 marks]**What should be the value of $v_B$ if the shuttle's trajectory is to be tangent at $A$ to the orbit of the trajectory?

2. **[20 marks]**The mechanism shown is used to raise a worker to the elevation of overhead electric and telephone wires. The entire mechanism rotates at the constant rate $\omega_1 = 0.15$ $rad/s$ about the $y'$ axis. The frame $O'x'y'z'$ is attached to the mechanism and rotates with it. The angle between arm $O'B$ and the horizontal is constant, while arm $BC$ is being lowered at the constant rate $\omega_2 = 0.20$ $rad/s$. The arms $O'B$ and $BC$ are each 4.5 $m$ long. Determine the acceleration of point $C$ at the instant shown in terms of the unit vectors of the $O'x'y'z'$ frame.



3. [**20 marks**]A thin homogeneous disk of mass $m$ and radius $r$ is rigidly mounted on the horizontal axle $AB$. The plane of the disk forms an angle $\beta$ with the vertical and the axle rotates with angular speed $\omega$ about the $x$ axis. Determine the angular momentum of the disk about its centre in the $x-y-z$ coordinate frame.




4. [**20 marks**]A light elastic spring of stiffness $k$ is clamped at its upper end and supports a particle of mass $m$ at its lower end. A second spring of stiffness $k$ is fastened to the particle and, in turn, supports a particle of mass $2m$ at its lower end. The equations of motion are

$$\begin{aligned} m\ddot{x}_1 + 2kx_1 - kx_2 &= 0 \\ 2m\ddot{x}_2 - kx_1 + kx_2 &= 0 \end{aligned}$$

where $x_1, x_2$ are the displacements of the masses from the static equilibrium positions. Determine the natural frequencies $\omega_k$ , normal mode vectors $\{s\}_k$ and normal coordinates $r_k$ such that the equations of motion may be written in the de-coupled form

$$\ddot{r}_k + \omega_k^2 r_k = 0 \qquad (k = 1, 2)$$

5. [**20 marks**]A particle $P$ of mass $m$ is fixed to the centre of a weightless rod of length $a$ and the ends of the rod are constrained to move along smooth walls as shown. Using generalised coordinate $\theta$ as shown, determine Hamilton's equations of motion and find the rod's angular acceleration when $\theta = 30^0$.