Physics 3150

Final Examination

15 April 2005

Allotted time: 120 minutes.

Work four questions out of the following six. All questions are of equal value.

1. Explain what is meant by the "Fermi energy" ε_F of a degenerate ideal gas of fermions. For spin 1/2 fermions it is given by

$$\varepsilon_F = \frac{\hbar^2 \pi^2}{2m} \left(\frac{3}{\pi} \frac{N}{V} \right)^{2/3} .$$

Show that the internal energy of such a gas is given by $U = (3/5)\, N\varepsilon_F$. Find its pressure in terms of N and V.

   (a) Describe the size, mass, and composition of a typical white dwarf. Give an example of a white dwarf.

2. (a) Sketch the structures of late, middling (G, say), and early Main Sequence stars, indicating the positions and extent of their cores, radiative zones, and convective zones, and coronas.

   (b) Derive the Equation of Hydrostatic Support.
   If the density of a star of radius R_* and mass M_* were *constant*, find its pressure $P(r)$ as a function of radius r, and show that its central pressure P_c is proportional to GM_*^2/R_*^4.

3. Write down the reactions of the pp I chain and of the main CNO cycle, Which of the fundamental forces are involved in each reaction?

   (a) Late main sequence stars are powered primarily by the pp chains while early main sequence stars are powered mainly by the CNO cycles. Explain why, for material of solar composition, the pp chains produce more power than the CNO at lower temperatures while the CNO cycles produce more power than the pp chains at higher temperatures.

   (b) State one set of reactions which leads from ${}^4_2\mathrm{He}$ to ${}^{12}_6\mathrm{C}$.

4. Sketch the Hertzsprung–Russell diagram, showing the location of the Main Sequence, white dwarfs, subgiants, giants, and supergiants.

   (a) State the spectral classification and attempt to estimate the luminosity classes of the following two stars, and locate them on the Hertzsprung–Russell diagram.

      i. Star 1 is red. Neutral metal absorption lines, particularly of Ca I at $\lambda 4226$ and of Fe I at $\lambda 4045$ are strong. The Balmer lines are not observable. Molecular bands are conspicuous. The lines are relatively broad.

      ii. Star 2 has weak Balmer and He I absorption lines, and He II *emission* lines. The lines are all very narrow. The colour of the star is blue-white; its peak emission is in the near UV.

   (b) How does the spectrum of an A0 star differ from that of a perfect black body of the same T_e?

5. Describe carefully the post–Main Sequence evolution of a high–mass star, making reference to the role of the virial theorem. Show cross sections of the star at various stages of its evolution. State as many of the relevant nuclear reactions as you can.

   (a) How does the post–Main Sequence evolution of a high–mass star differ from that of a low–mass star?

6. (a) State the typical sizes and masses for white dwarfs and neutron stars.

   (b) Outline the processes by which energy is liberated from accretion in contact and semi–detached binary systems in which one component is compact. Mention accretion disks and mass transfer streams, etc.

   (c) If the compact component is a white dwarf, describe briefly the "cataclysmic variable" behaviour exhibited by such systems. How are white dwarfs involved in Type I supernovas?

Formula Sheet for P3150 Final Examination

You may use any of the following:

$$\begin{aligned} c &= 3.0 \times 10^{8}\,\mathrm{m}\cdot\mathrm{s}^{-1} = 3.0\times 10^{10}\,\mathrm{cm}\cdot\mathrm{sec}^{-1} \\ \hbar &= 1.1\times 10^{-34}\,\mathrm{J}\cdot\mathrm{s} = 1.1\times 10^{-27}\,\mathrm{erg}\cdot\mathrm{sec} \\ k_B &= 1.4\times 10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1} = 1.4\times 10^{-16}\,\mathrm{erg}\cdot\mathrm{K}^{-1} \end{aligned}$$

$$\varepsilon_{n_x n_y n_z} = \frac{\hbar^2\pi^2}{2m}\left\{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right\}$$

$$\begin{aligned} m_p &= 938.27231\,\mathrm{MeV} \simeq 938\,\mathrm{MeV} = 1.7\times 10^{-24}\,\mathrm{gm} \\ m_n &= 939.56563\,\mathrm{MeV} \simeq 940\,\mathrm{MeV} \\ m_e &= 0.51099906\,\mathrm{MeV} \simeq 0.511\,\mathrm{MeV} \\ m_\mu &= 106\,\mathrm{MeV} \\ m_\tau &= 1784\,\mathrm{MeV} \\ m_{\pi^0} &= 135\,\mathrm{MeV} \\ m_{\pi^\pm} &= 140\,\mathrm{MeV} \end{aligned}$$

$$\begin{aligned} a &= \frac{8\pi^5 k_B^4}{15h^3c^3} = 76\,\mathrm{kPa}\cdot\left(10^5\,\mathrm{K}\right)^{-4} \\ \sigma &= \frac{2\pi^5 k_B^4}{15h^3c^2} = 5.7\,\mathrm{W}\cdot\mathrm{m}^{-2}\cdot(100\,\mathrm{K})^{-4} \\ B_\lambda &= \frac{2hc^2/\lambda^5}{e^{hc/\lambda k_B T}-1} \\ B_\nu &= \frac{2h\nu^3}{c^2\left(e^{hc/\lambda k_B T}-1\right)} \end{aligned}$$

$$\begin{aligned} B - V &\simeq 7200/T_e - 0.64 \\ M_V &\simeq 29500/T_e - 5\log_{10}(R/R_\odot) - 0.08 \\ M_V - V &= -5\log_{10}(d/10\,\mathrm{pc}) \end{aligned}$$

$$\begin{aligned} \frac{dP(r)}{dr} &= -G\frac{M(r)\rho(r)}{r^2} \quad \text{equation of hydrostatic support} \\ \frac{dM(r)}{dr} &= 4\pi r^2\rho(r) \quad \text{equation of continuity} \\ \frac{dL(r)}{dr} &= 4\pi r^2\varepsilon(r)\rho(r) \quad \text{luminosity equation} \\ \frac{dT(r)}{dr} &= -\frac{3\overline{\kappa}(r)\rho(r)}{4acT(r)^3}\frac{L(r)}{4\pi r^2} \quad \text{equation of radiative transport} \end{aligned}$$

$$dU = TdS - PdV.$$