Name:______________________________
Student number:____________________

DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

Final Exam **Physics 3000** **December 11, 2010**

Fall 2010 **9:00-11:00**

INSTRUCTIONS:

1. Answer all seven (7) questions. The marks for each question are indicated. The total number of marks on the paper is 100. Budget time accordingly.
2. Write answers in the spaces provided. If additional space is needed, use the backs of sheets but indicate clearly where your answer is continued.
3. Where appropriate, units must be included for answers to be considered complete.
4. A list of useful formulae, a table of constants, and tables of semiconductor properties are attached at the end of the paper (last three pages). For your convenience, three particularly useful parameters are listed below.

Particularly useful parameters:

Relative permittivity (dielectric constant) for silicon: $\epsilon_{\mathrm{Si}} = 11.7$

Relative permittivity (dielectric constant) for silicon oxide: $\epsilon_{\mathrm{SiO_2}} = 3.9$

Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-14}\ \mathrm{F/cm}$

Intrinsic carrier concentration for silicon at 300 K: $n_{i,\mathrm{Si}} = 1.5 \times 10^{10}\ \mathrm{cm^{-3}}$

Thermal voltage $\left(V_t = \frac{kT}{e}\right)$ at 300 K: $V_t(300\ \mathrm{K}) = 0.0259\ \mathrm{V}$

Electron charge: $e = 1.6 \times 10^{-19}\ \mathrm{C}$

Planck's constant: $h = 6.625 \times 10^{-34}$ J-s = 4.135×10^{-15} eV-s

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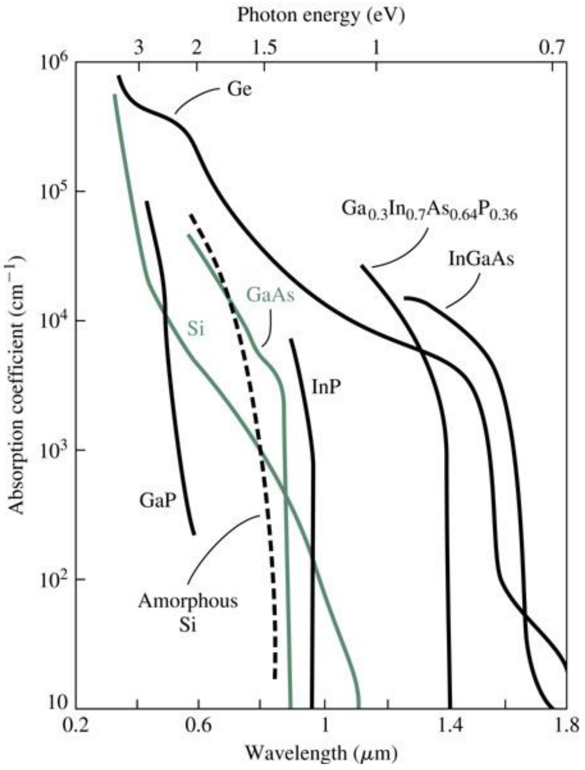
1. [15 marks]

(a) (6 marks) Assume that a sample of Ge is illuminated by light with a wavelength $\lambda = 1.1\ \mu$m.

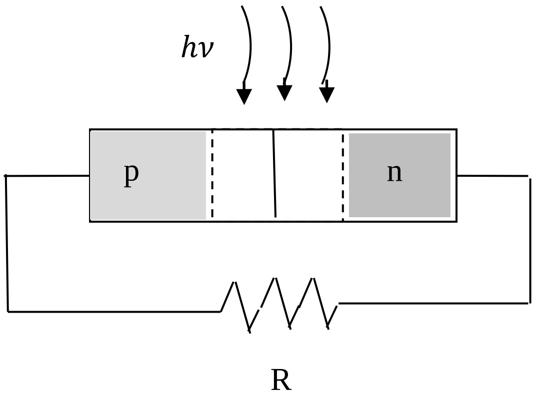
(i) Calculate the photon energy. Give your answer in Joules.

(ii) Calculate the electron-hole pair generation rate at a depth in Germanium where the photon intensity is $I_\nu(x) = 0.06\ \mathrm{W/cm^2}$? Use the graph to estimate the relevant absorption coefficient.



(a) The diagram shows a pn junction solar cell connected to a resistive load.

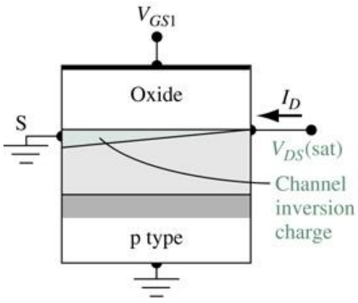
(i) (4 marks) On the diagram, indicate and label the directions of the photocurrent, I_L, and the forward-bias current, I_F. Briefly explain why the **photocurrent** has the direction you've shown.



(ii) (5 marks) A silicon pn junction solar cell is illuminated so that the **photocurrent** is $I_L = 0.42$ A and the voltage across the load resistor is found to be $V = 0.57$. The diode reverse saturation current is $I_S = 3.6 \times 10^{-11}$ A. What is the **total** current through the diode? Assume $T = 300$ K.

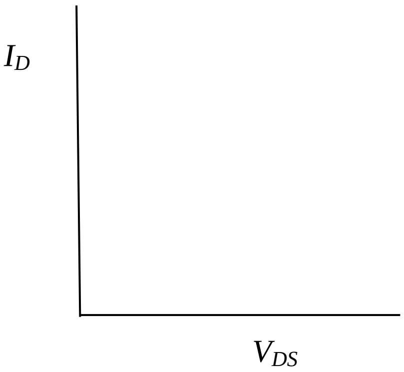
2. [14 marks]

(a) (4 marks) The diagram to the right shows an n-channel enhancement mode MOSFET for which the drain-source voltage is $V_{DS} = V_{DS}(\text{sat})$. Briefly explain why channel thickness goes to zero at the drain for this value of V_{DS}.



(c)

(b) (5 marks) Using the axes to the right, sketch and label curves showing how the drain current, I_D, in this device depends on V_{DS} for two values of the gate source potential difference, V_{GS1} and V_{GS2} where $V_{GS2} > V_{GS1}$ and $V_{GS1} > V_T > 0$. Identify $V_{DS}(\text{sat})$ on your curves.



(c) (5 marks) Assume that the threshold voltage for a particular n-channel enhancement MOSFET is 0.65 V and that it is biased in the saturation region (i.e. $V_{DS} > V_{DS}(\text{sat})$). When $V_{GS} = 2.5$ V, the drain current is found to be $I_D = 3.2$ mA. What is the drain current for $V_{GS} =$ 5.0 V?

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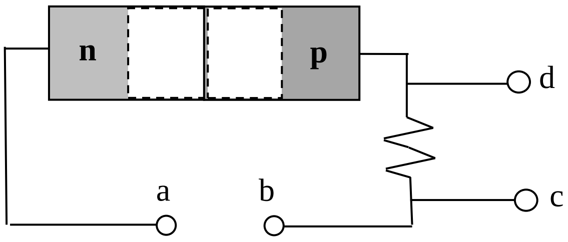
3. [16 marks]

(a) The drawing to the right represents a pn junction diode operated as a photodiode.

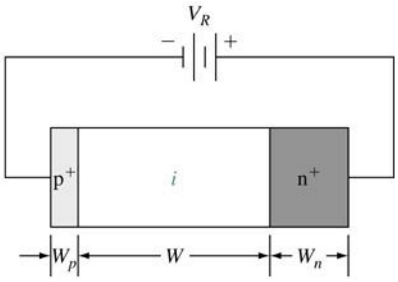


(i) (3 marks) On the diagram, indicate the direction of the electric field induced by ionized impurities in the space charge region.

(ii) (3 marks) On the diagram, indicate which of the biasing terminals (a or b) should be positive and which should be negative in order for the junction to function as a photodiode. Also indicate the direction of the current that flows when the junction is illuminated.

(iii) (4 marks) Briefly explain the difference between prompt photocurrent and delayed photocurrent. In which regions of the device are the carriers contributing to the prompt and delayed photocurrent generated?

(b) (6 marks) The picture to the right shows a reverse-biased PIN photodiode.

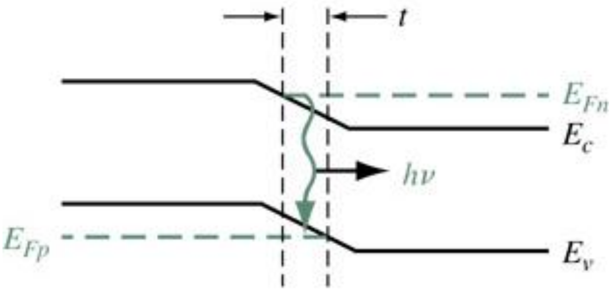


(i) Briefly explain why the space charge region is said to extend completely through the intrinsic region.

(ii) The intrinsic region increases photodetector sensitivity. Does it affect prompt photocurrent or delayed photocurrent? Briefly explain your answer.

4. [10 marks]

(a) (6 marks) The energy-band diagram to the right represents a forward-biased degenerately doped pn junction.

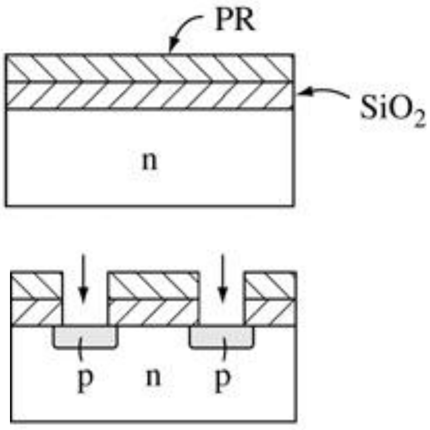


(i) Briefly explain what aspect of this diagram shows that the p-type and n-type materials in this device are degenerately doped.

(ii) Briefly explain why there is a population inversion (i.e. higher concentration of electrons in the conduction band than in the valence band) in the central region of the junction (labeled as thickness *t*).

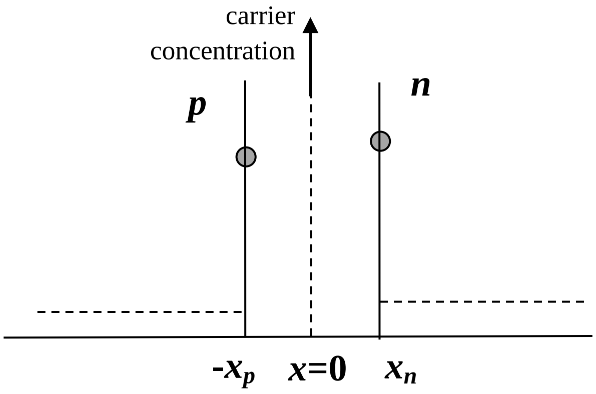
(iii) What kind of device depends on the presence of a population inversion for its operation?

(b) (4 marks) The pictures to the right show two stages in the fabrication of pn junction diodes. In the top picture, a layer of photoresist has been applied over an oxide layer previously grown on the surface of an n-type silicon substrate. The bottom picture shows the material after two regions of p-type Silicon have been formed. **Briefly** describe the fabrication steps between the stage shown in the top picture and the stage shown in the bottom picture.



5. [16 marks]

(a) The dots on this diagram represent minority-carrier concentrations at the space charge edges for a forward-biased pn junction. The horizontal dashed lines represent thermal equilibrium minority-carrier concentrations.



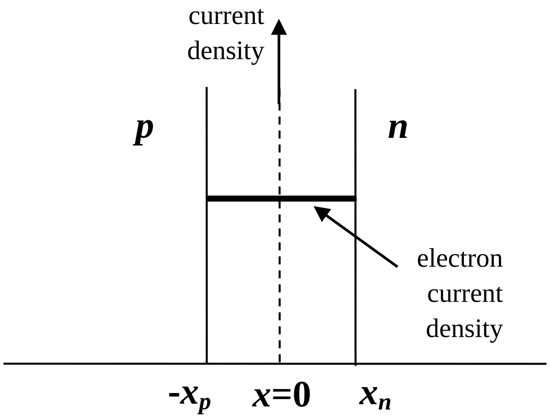
(i) (2 marks) Attach the labels n_{p0}, p_{n0}, $n_p(-x_p)$, and $p_n(x_n)$ to the appropriate features on the diagram.

(ii) (3 marks) Assume that the semiconductor is Si and that the impurity concentrations on the p-type side and n-type side, respectively, are $N_a = 5.0 \times 10^{16}$ cm^{-3} and $N_d = 2.0 \times 10^{16}$ cm^{-3}. Calculate the **thermal equilibrium minority-carrier** concentration on the **p-type** side assuming complete impurity ionization and $T = 300$ K.

(iii) (3 marks) Calculate the **minority-carrier** concentration at the **space charge edge** on the p-type side, $x = x_p$, if a forward-bias of 0.6 V is applied to this junction.

(iv) (4 marks) On the diagram above, draw and label curves to represent the steady-state minority-carrier concentrations in this device under forward bias conditions. Briefly explain why these concentrations depend on position in the way you have shown in your diagram.

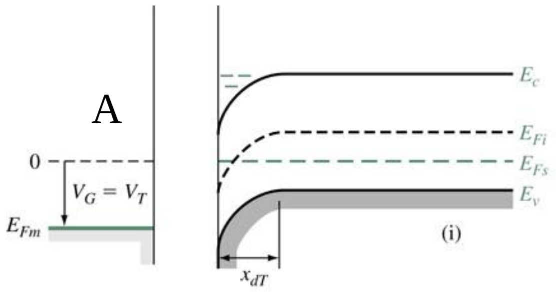
(b) (4 marks) The diagram to the right shows the electron contribution to **current density** in the space charge region of a forward-biased pn junction. Draw and label curves representing the corresponding majority-carrier electron current density and minority-carrier electron current density in the appropriate regions.



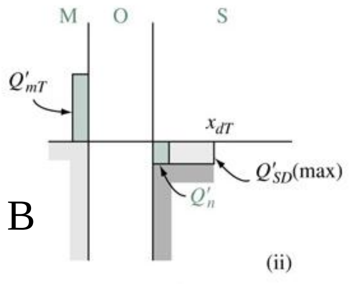
6. [14 marks]

Pictures A and B show the energy bands and charge distribution for a MOS capacitor at **threshold** gate voltage.

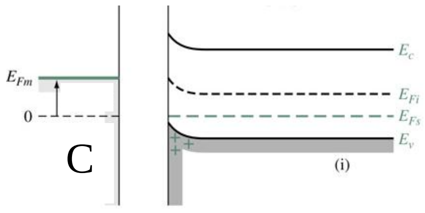
(a) (3 marks) Is this energy band diagram more characteristic of an **n-channel** or a **p-channel device**? Briefly explain your answer.

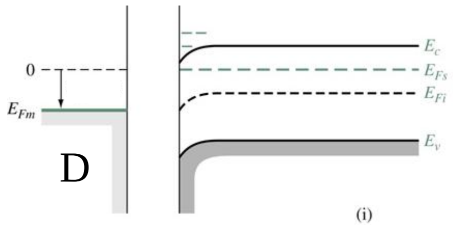


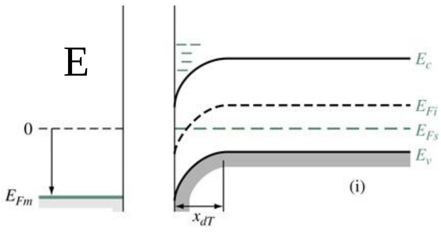
(b) (4 marks) **Briefly** explain/describe the three charge densities, Q'_{mT}, Q'_n, and Q'_{SD}(max), shown in figure B.



(c) (4 marks) Which of the diagrams below (C, D, or E) best represents the energy bands for this MOS capacitor when it is biased into the inversion state? **Briefly** justify your choice and write an inequality showing how the gate voltage and the threshold voltage are related for this condition?







(d) (3 marks) Draw a diagram showing the charge distribution for this MOS capacitor in the inversion state. Label your diagram as completely as possible.

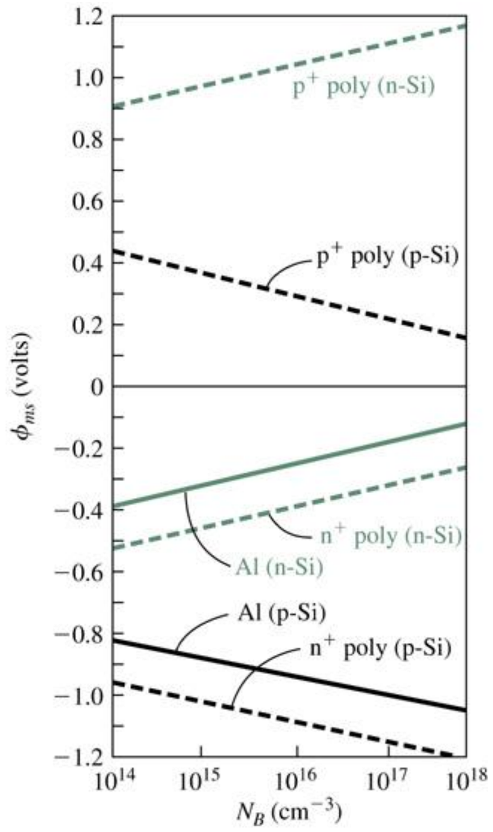
7. [15 marks] A MOS transistor is fabricated on a p-type silicon substrate with $N_a = 2 \times 10^{16}\text{cm}^{-3}$. The oxide thickness is $t_{\text{ox}} = 450$ Å. The equivalent fixed oxide charge is $Q'_{ss} = 5 \times 10^{10}$ electronic charges/cm^2. The relative permittivities for Si and SiO$_2$ are given on the front page of the exam. Assume $T = 300$K.

(a) (3 marks) What is the potential difference $\phi_{Fp} = E_F - E_{Fi}$ in the semiconductor assuming complete ionization?

(b) (3 marks) What is the maximum space charge width, x_{dT}, in the semiconductor?

(c) (3 marks) What is the magnitude of the maximum space charge density per unit area, $|Q'_{SD}(\text{max})|$, in the depletion region of the semiconductor?

(d) (6 marks) Assuming that the gate material is aluminum, find ϕ_{ms} from the graph of ϕ_{ms} versus impurity concentration provided to the right and calculate the **threshold voltage** for this device. (Hint: note that Q'_{ss} is given in units of electronic charges/cm^2)



Potentially useful formulae

Photons: $E_{\text{photon}} = h\nu$; $p = h/\lambda$; $c = \nu\lambda$; Bohr Atom: $E_n = -\frac{m_0 e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2}$; $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_0 e^2}$

Drift curr. density: $J_n = -e\sum_{i=1}^{n} v_i$; Eff. mass: $F_{\text{ext}} = m^* a$; Free particle: $E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

Densities of states: $g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$; $g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$

F-D prob. function: $f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$; M-B approx.: $f_F(E) \approx \exp\left[-\frac{(E-E_F)}{kT}\right]$

Carrier distributions: $n(E) = g_c(E) f_F(E)$; $p(E) = g_v(E)[1 - f_F(E)]$

Effective densities of states: $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$; $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

Therm. equil. carrier concentrations.: $n_0 = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$; $p_0 = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$

Intrins. carr. concentration & Fermi level: $n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$; $E_{Fi} - E_{\text{midgap}} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$

Extrinsic semiconductor relationships: $n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$; $p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$

Fundamental semiconductor equation: $n_0 p_0 = n_i^2$

Donor electron relationships: $n_d = N_d - N_d^+ = \frac{N_d}{1+\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$; $\frac{n_d}{n_d + n_0} = \frac{1}{1+\frac{N_c}{2N_d}\exp\left[\frac{-(E_c - E_d)}{kT}\right]}$

Acceptor hole relationships: $p_a = N_a - N_a^- = \frac{N_a}{1+\frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)}$; $\frac{p_a}{p_a + p_0} = \frac{1}{1+\frac{N_v}{gN_a}\exp\left[\frac{-(E_a - E_v)}{kT}\right]}$

Compensated semiconductors, complete ionization:

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \qquad p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Fermi energy level relationships:

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_0}\right) \; ; \quad E_F - E_v = kT\ln\left(\frac{N_v}{p_0}\right) \; ; \quad E_F - E_{Fi} = kT\ln\left(\frac{n_0}{n_i}\right) = -kT\ln\left(\frac{p_0}{n_{\text{i}}}\right)$$

Conduct. & resist.: $e(\mu_n n + \mu_p p) = \sigma = \frac{1}{\rho}$ Mobility: $\mu_p = \frac{v_{dp}}{\varepsilon} = \frac{e\tau_{cp}}{m_p^*}$; $\mu_n = \frac{v_{dn}}{\varepsilon} = \frac{e\tau_{cn}}{m_n^*}$

Drift curr. dens.: $J_{drf} = e(\mu_n n + \mu_p p)\varepsilon_x$ Diffusion curr. dens.: $J_{dif} = eD_n\frac{dn}{dx} - eD_p\frac{dp}{dx}$

Total current density: $J = J_{drf} + J_{dif}$ Einstein relation: $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$

Graded impurity distribution: $\varepsilon_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$

Equil. generation & recombination rates: $G_{n0} = G_{p0} = R_{n0} = R_{p0}$; $R_{n0} = \frac{n_0}{\tau_{n0}}$; $R_{p0} = \frac{p_0}{\tau_{p0}}$

Excess carrier recombination (p-type): $\delta n(t) = \delta n(0)e^{-t/\tau_{n0}}$; $R_n' = R_p' = \frac{\delta n(t)}{\tau_{n0}}$; $\tau_{n0} = \frac{1}{\alpha_r p_0}$

Excess carrier recombination (n-type): $\delta n(t) = \delta n(0)e^{-t/\tau_{p0}}$; $R_n' = R_p' = \frac{\delta n(t)}{\tau_{p0}}$; $\tau_{p0} = \frac{1}{\alpha_r n_0}$

Hall effect: $\vec{F} = q[\vec{\varepsilon} + \overrightarrow{v_d} \times \vec{B}]$; $p = \frac{I_x B_z}{edV_H}$; $n = -\frac{I_x B_z}{edV_H}$; $\mu_p = \frac{I_x L}{epV_x Wd}$; $\mu_n = \frac{I_x L}{enV_x Wd}$

Built in potential barrier (pn junction): $V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right)$

Potentially useful formulae (continued)

Poisson's equation: $\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_s} = -\frac{d\varepsilon(x)}{dx}$

Sp. charge width: $x_n = \left[\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a+N_d}\right]^{1/2}$; $x_p = \frac{N_d}{N_a}x_n$; $W = \left[\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$

Electric field: $\varepsilon_{\text{max}} = -\frac{2(V_{bi}+V_R)}{W}$ Junction Capacitance: $C' = \left[\frac{e\epsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$

Schottky barrier: $\phi_{B0} = (\phi_m - \chi)$ $V_{bi} = \phi_{B0} - \phi_n$

pn diode: $J_D = J_S\left[\exp\left(\frac{eV_D}{kT}\right) - 1\right]$

Schottky barrier diode: $J = \left[A^*T^2\exp\left(\frac{-\phi_{B0}}{kT}\right)\right]\left[\exp\left(\frac{eV_D}{kT}\right) - 1\right] = J_{ST}\left[\exp\left(\frac{eV_D}{kT}\right) - 1\right]$

MOS depletion width (p-type): $x_{dT} = \left(\frac{4\epsilon_s|\phi_{Fp}|}{eN_a}\right)^{1/2}$; $e\phi_{Fp} = E_F - E_{Fi} = -kT\ln\left(\frac{N_a}{n_i}\right)$

MOS depletion width (n-type): $x_{dT} = \left(\frac{4\epsilon_s\phi_{Fn}}{eN_d}\right)^{1/2}$; $e\phi_{Fn} = E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

Metal-semiconductor work function difference (p-type): $\phi_{ms} = \left[\phi_m' - \left(\chi' + \frac{E_g}{2e} + |\phi_{Fp}|\right)\right]$

Metal-semiconductor work function difference (n-type): $\phi_{ms} = \left[\phi_m' - \left(\chi' + \frac{E_g}{2e} - \phi_{Fn}\right)\right]$

Flat-band voltage: $V_{FB} = \phi_{ms} - \frac{Q_{ss}'}{C_{\text{ox}}}$ Maximum space charge density: $|Q_{SD}'(\text{max})| = eN_a x_{dT}$

Threshold voltage: $V_{TN} = \frac{|Q_{SD}'(\text{max})|}{C_{\text{ox}}} - \frac{Q_{ss}'}{C_{\text{ox}}} + \phi_{ms} + 2|\phi_{Fp}|$

MOSFET (general): $I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$; $K_n = \frac{W}{L}\frac{\mu_n C_{\text{ox}}}{2}$; $C_{\text{ox}} = \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}}$

MOSFET (saturation): $V_{DS}(\text{sat}) = V_{GS} - V_T$; $I_D(\text{sat}) = K_n(V_{GS} - V_{TN})^2$

Electron inversion charge density: $n_s = N_a\exp\left(\frac{\Delta\phi_s}{V_t}\right)$

Channel length mod.: $\Delta L = \sqrt{\frac{2\epsilon_s}{eN_a}}\left[\sqrt{|\phi_{FP}| + V_{DS}(\text{sat}) + \Delta V_{DS}} - \sqrt{|\phi_{FP}| + V_{DS}(\text{sat})}\right]$; $\frac{I_D'}{I_D} = \frac{L}{L-\Delta L}$

Mobility variation: $\varepsilon_{\text{eff}} = \frac{1}{\epsilon_S}\left(|Q_{SD}'(\text{max})| + \frac{1}{2}Q_n'\right)$; $\mu_{\text{eff}} = \mu_0\left(\frac{\varepsilon_{\text{eff}}}{\varepsilon_0}\right)^{-1/3}$

Forward-biased pn junction.:

minority carrier conc. at space charge edge: $n_p = n_{p0}\exp\left(\frac{V_a}{V_t}\right)$; $p_n = p_{n0}\exp\left(\frac{V_a}{V_t}\right)$

reverse saturation current density: $J_S = \left(\frac{e\sqrt{D_p}p_{n0}}{\sqrt{\tau_{p0}}} + \frac{e\sqrt{D_n}n_{p0}}{\sqrt{\tau_{n0}}}\right)$

common-base current gain: $\alpha = \frac{i_C}{i_E}$ common-emitter current gain: $\beta = \frac{i_C}{i_B}$

photon flux intensity: $I_\nu(x) = I_{\nu 0}e^{-\alpha x}$ electron-hole pair generation rate: $g' = \frac{\alpha I_\nu(x)}{h\nu}$

pn junct. solar cell: $I = I_L - I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$; $V_{\text{oc}} = V_t\ln\left(1 + \frac{I_L}{I_S}\right)$; $\left(1 + \frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$

photoconductor: $I_L = eG_L\tau_p(\mu_n + \mu_p)A\varepsilon$ photoconductor gain: $\Gamma_{\text{ph}} = \frac{\tau_p\varepsilon}{L}(\mu_n + \mu_p)$

pn photodiode prompt photocurrent: $J_{L1} = eG_L W$

PIN photodiode prompt photocurrent: $J_L = e\Phi_0(1 - e^{-\alpha W})$

Constants and Conversions

$$N_A = 6.02 \times 10^{23}$$

$$k = 1.38 \times 10^{-23}\text{ J/K} = 8.62 \times 10^{-5}\text{ eV/K}$$

$$e = 1.60 \times 10^{-19}\text{ C}$$

$$m_0 = 9.11 \times 10^{-31}\text{ kg}$$

$$\epsilon_0 = 8.85 \times 10^{-14}\text{ F/cm}$$

$$h = 6.625 \times 10^{-34}\text{ J-s} = 4.135\times10^{-15}\text{ eV-s}$$

$$c = 2.998 \times 10^{10}\text{ cm/s}$$

$$1\text{ eV}=1.6\times10^{-19}\text{ J}$$

Silicon, gallium arsenide, and germanium properties at T = 300 K

Property	Si	GaAs	Ge
Atoms/cm^3	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (^{o}C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Electron mobility, μ_n (cm^2/V-s)	1350	8500	3900
Hole mobility, μ_p (cm^2/V-s)	480	400	1900
Electron effective mass (DoS) $\left(\frac{m_n^*}{m_0}\right)$	1.08	0.067	0.55
Hole effective mass (DoS) $\left(\frac{m_p^*}{m_0}\right)$	0.56	0.48	0.37

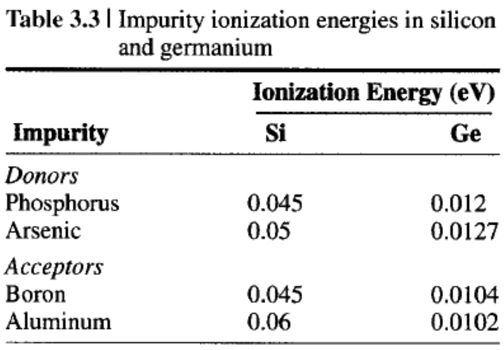


Table 3.3 | Impurity ionization energies in silicon and germanium

Impurity	Ionization Energy (eV)	
	Si	Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

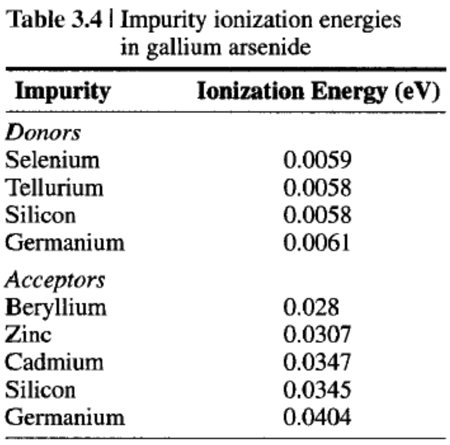


Table 3.4 | Impurity ionization energies in gallium arsenide

Impurity	Ionization Energy (eV)
Donors	
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptors	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404