Physics 4850, Final examination
9am-11am, Wednesday, December 8, 2004
Instructor: S. H. Curnoe

Instructions: Do all questions. Calculators not permitted.

1. Suppose a system is in the state

$$|\psi\rangle = \frac{1}{N}(|3,3;\uparrow\rangle + i|2,2;\downarrow\rangle - \sqrt{3}|2,0;\downarrow\rangle),$$

where the kets are angular momentum and spin eigenstates $|l, m; \pm 1/2\rangle$.

   (a) What is the normalisation condition on N?
   (b) Write $|\psi\rangle$ as a real space function.
   (c) What is the expectation value (average) for L_y in the state $|\psi\rangle$?
   (d) If S_x is measured. What values may be found, with what probabilities?
   (e) Suppose L^2 is measured and found to be $6\hbar^2$. What is the state of the system immediately after the measurement? Your answer should be normalised.

2. Addition of Angular Momenta

   (a) What is the dimension of the space for a particle with $S = 1/2$ and $L = 1$?
   (b) What are the allowed values of the total angular momentum J?
   (c) Let $|l, m_l; s, m_s\rangle$ be an eigenket of L^2, L_z, S^2, S_z and $|j, m_j\rangle$ be an eigenket of J^2 and J_z. Write down a set of basis kets which span the space of $S = 1/2$ and $L = 1$ using the kets
      i. $|l, m_l; s, m_s\rangle$ and
      ii. $|j, m_j\rangle$
   (d) Find the Clebsch-Gordan coefficients which relate the bases in (c).

3. Perturbation theory
Suppose $H_0 = (\vec{L}^2 + \vec{S}^2)/\mu + \mu_B B(2S_z + L_z)$. The eigenstates are $|l, m_l; s, m_s\rangle$.

(a) What are the eigenvalues?

The system is perturbed by $\lambda\vec{L}\cdot\vec{S}$, where λ is a small, real constant.

(b) Show that

$$\vec{L}\cdot\vec{S} = L_z S_z + \frac{1}{2}(L_+ S_- + L_- S_+).$$

(c) What is

$$(\vec{L}\cdot\vec{S})|2,2;1/2,-1/2\rangle?$$

(d) Therefore what are the corrections for the state $|2,2;1/2,-1/2\rangle$

i. to second order in λ for the eigenvalue,

ii. and to first order in λ for the eigenket?

4. Time evolution

Let $|\phi_n\rangle$ be an eigenket of the harmonic oscillator Hamiltonian, $H_0 = \hbar\omega_0(a^\dagger a + 1/2)$ such that $H_0|\phi_n\rangle = \hbar\omega_0(n+1/2)|\phi_n\rangle$.

(a) If a system is in the ground state $|\psi(0)\rangle = |\phi_0\rangle$ at $t = 0$, what is $|\psi(t)\rangle$?

(b) Now suppose that a small perturbation is added to the Hamiltonian, $H = H_0 + W(t)$, $W(t) = 0$ for $t < 0$ and $W(t) = \lambda(a^\dagger + a)\sin\omega t$ for $t > 0$.

i. For which n are the matrix elements $\langle\phi_n|W(t)|\phi_0\rangle$ non-zero?

ii. For what value of ω will the system be in resonance?

iii. When the system is in resonance, does it absorb energy or emit energy?

5. Thought Questions

(a) List three Hamiltonians (words, not equations) for single particles in real space potentials which you can now solve exactly.

(b) List two non-zero commutators (which therefore correspond to observables that you cannot measure simultaneously).

(c) List two sets of two observables which can be measured simultaneously.

(d) What's so special about angular momentum in quantum mechanics (two points)?

Useful formulae:

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$J_\pm|j,m\rangle = \sqrt{(j(j+1) - m(m\pm 1)}|j,m\pm 1\rangle$$

Harmonic Oscillator:

$$a|\phi_n\rangle = \sqrt{n}|\phi_{n-1}\rangle$$

$$a^\dagger|\phi_n\rangle = \sqrt{n+1}|\phi_{n+1}\rangle$$

Non-degenerate perturbation theory:

$$E_n = E_n^0 + \lambda\langle\phi_n|W|\phi_n\rangle + \lambda^2\sum_{m\neq n}\frac{|\langle\phi_n|W|\phi_m\rangle|^2}{E_n^0 - E_m^0} + o(\lambda^3)$$

$$|\psi_n\rangle = |\phi_n\rangle + \lambda\sum_{m\neq n}\frac{|\phi_m\rangle\langle\phi_m|W|\phi_n\rangle}{E_n^0 - E_m^0} + o(\lambda^2)$$