Memorial University
Department of Physics and Physical Oceanography
Physics 4500

Winter 2007

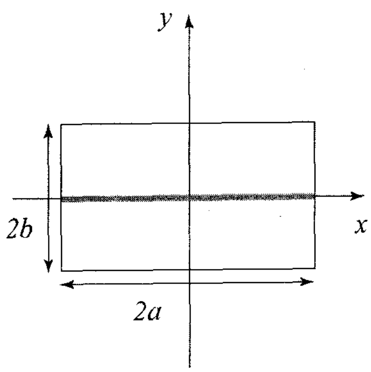
Final Exam: Time 2 hours

Instructions:

- Do any three of the four questions.
- Clearly state any assumptions made in your derivations.
- You may refer to the text by Griffiths, but no other books or notes are permitted.

Question 1: Consider an infinitely long conducting rectangular box with sides of length $2a$ and $2b$ with $V = 0$. The coordinate axis is chosen so the box is aligned along the z-axis with it's sides along the x and y axis, as shown below. Inside the box, charge is distributed uniformly across the plane $x = 0$, with a surface charge density σ. The potential inside the box may be written as

$$V(x,y) = \begin{cases} \sum_{n=0}^{\infty} A_n \cos(k_n x)\sinh(k_n(b-y)) & y > 0 \\ \sum_{n=0}^{\infty} B_n \cos(k_n x)\sinh(k_n(b+y)) & y < 0 \end{cases}$$



a. Explain briefly how are the coefficients k_n are determined and derive an expression for them.
b. Explain briefly how the coefficients A_n and B_n are determined and derive an expression for them.

Hint: $\frac{1}{a}\int_{-a}^{a} dx \cos(k_m x)\cos(k_n x) = \delta_{mn}$

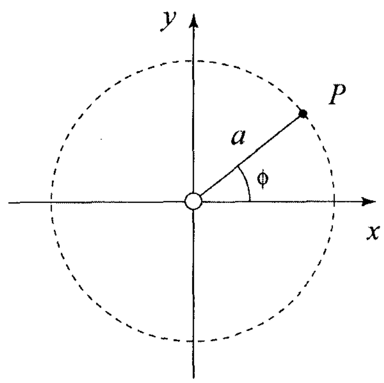
Question 2: A current I flows down a long straight wire of radius a. The current is distributed uniformly across the wire and the wire is made from a material with a magnetic susceptibility χ_m.

a. Calculate the $\vec{H}$ both inside ($\rho < a$) and outside ($\rho > a$) the wire. State any assumptions made in your derivation.
b. Calculate both $\vec{B}$ and $\vec{M}$ inside and outside the wire.
c. Determine the bound currents both inside the wire and on the surface.
d. What is the total bound current flowing down the wire.

Question 3: The magnetic field due to an infinitely long wire lying along the z-axis carrying a current I in the positive z-direction, in a uniform magnetic field of magnitude B_0, aligned parallel to the y-axis, is given by

$$\vec{B} = B_0\hat{y} - \frac{\mu_0 I}{2\pi\rho}\left(\sin\phi\hat{x} - \cos\phi\hat{y}\right)$$

a. Calculate all the non-vanishing components of the Maxwell stress tensor at a point P a distance a from the wire.
b. Calculate F_x acting on the volume V enclosed by a cylinder of length L and radius a from the integral $\oint_S \vec{T}\cdot d\vec{a}$ where S denotes the surface of a cylinder.
c. Without performing any further calculation what would you expect the y component of the force acting on the volume V to be. Explain your reasoning.



Hint: $\int_0^{2\pi}\cos^3\phi\, d\phi = \int_0^{2\pi}\sin^3\phi\, d\phi = 0$

$\int_0^{2\pi}\cos^2\phi\, d\phi = \int_0^{2\pi}\sin^2\phi\, d\phi = \pi$

Question 4: Inside a conductor the Maxwell's equations admit plane wave solutions of the form

$$\vec{E} = \tilde{E}_0 e^{i\left(\tilde{k}\cdot\vec{r} - \omega t\right)}$$

a. Show for a typical metal ($\sigma \sim 5\times10^7\ \mathrm{S\cdot m^{-1}}$) at optical frequencies ($\lambda \sim 5\times10^{-7}\ \mathrm{m}$) that $\tilde{k} = (1+i)/\delta$. Derive an explicit expression for the skin depth δ for a good conductor.
b. The complex amplitude of light with (angular) frequency ω reflected of a conductor at normal incidence is given by (Griffiths 9.141)

$$\tilde{E}_{0R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\tilde{E}_{0I}$$

where $\tilde{\beta} = c\tilde{k}/\omega$. Calculate $1 - R$ (R denotes the reflection coefficient) for a good conductor to leading order in δ/λ, where λ denotes the wavelength of the incident radiation.
c. If the intensity of light from the sun is approximately $10^3\ \mathrm{W/m^2}$ estimate the heat absorbed by a sheet of metal $1.0\ \mathrm{m^2}$ left out in the sun for 1 hour.

Note: Numerical calculations only require order of magnitude precision!