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PHYSICS 4300: Advanced Physical Oceanography
Final Exam
April 19, 2006

This is a **closed-book** exam.
You are allowed to use a calculator and a formula sheet.
You have **2** hours to answer **all** of the questions.

Pts.

1. The surface wind pattern over an idealized subtropical North Atlantic Ocean consists of easterlies between 15°N and 30°N and westerlies between 30°N and 45°N. Consider that the basin is rectangular with a meridional length $L_y = 3330$ km and a zonal length $L_x = 5000$ km, and that the wind stress components are *12*

$$\tau_x = -\tau_0 \cos\left(\frac{\pi y}{L_y}\right), \quad 0 \le y \le L_y,$$

$$\tau_y = 0, \quad 0 \le y \le L_y,$$

where $\tau_0 = 0.1$ N m^{-2}.
 a) Determine the wind-induced Ekman pumping w_s. What is its direction?
 b) Evaluate w_s at 30°N.
 c) Determine the sign of the relative vorticity of the geostrophic flow in the ocean interior, by using the assumption that the Ekman pumping induced by the bottom friction matches that induced by the wind.
 d) Calculate the vertical volume flux over the basin.
Take $\rho_0 = 1028$ kg m^{-3} and a constant value for the Coriolis parameter (evaluated at 30° N).

2. A barotropic Kelvin wave travels around New Zealand, covering a distance $L = 3000$ km in 12.4 hours. *6*
 a) What is the direction of propagation of the wave?
 b) Calculate the wave speed. What is the ocean depth that corresponds to this speed?
 c) Estimate the distance from shore over which the amplitude of the wave decreases by a factor $1/3$ (hint: $e \approx 2.7$). Consider a constant value for the Coriolis parameter $f = -10^{-4}$ s^{-1}.

Pts.

3. Consider a uniformly stratified ocean layer characterized by a buoyancy frequency $N = 3.5 \times 10^{-3}\ \mathrm{s}^{-1}$. *4*
   a) What is the minimum period of the internal waves that can be generated within the layer? (The wave period is $T = \frac{2\pi}{\omega}$, where ω is the wave frequency.)
   b) For an internal wave of frequency $\omega = \frac{N}{2}$, find the angle to the vertical at which seawater parcels are displaced by the wave.

4. Consider a southward geostrophic current in the Northern Hemisphere, whose speed decreases with depth and vanishes at the ocean bottom (assumed horizontal). *8*
   a) What are the force balances in the horizontal and vertical directions?
   b) Determine the horizontal pressure gradient at the bottom.
   c) Assuming that the surface meridional velocity is $v = -0.1\ \mathrm{m\ s}^{-1}$ calculate the slope of the sea surface in the zonal direction ($\frac{\partial \eta}{\partial x}$). Take $f = 10^{-4}\ \mathrm{s}^{-1}$.
   d) Make a rough sketch of a cross-section of the current showing the sea surface, a few isopycnals and a few isobars. Consider a uniform atmospheric pressure at the sea surface.

Total *30*

Solutions

1. a) The wind-induced Ekman pumping is

$$w_s = \frac{1}{\rho_0 f}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right).$$

We have

$$\frac{\partial \tau_x}{\partial y} = \frac{\tau_0 \pi}{L_y}\sin\left(\frac{\pi y}{L_y}\right) \quad \text{and} \quad \frac{\partial \tau_y}{\partial x} = 0.$$

Therefore,

$$w_s = -\frac{\tau_0 \pi}{\rho_0 f L_y}\sin\left(\frac{\pi y}{L_y}\right).$$

Since $\sin\left(\frac{\pi y}{L_y}\right) \geq 0$, for $0 \leq y \leq L_y$, and $f > 0$ in the Northern Hemisphere, we obtain $w_s < 0$, which implies downwelling over the entire basin.

b) At 30°N, $y = L_y/2$ and $f = 2\Omega \sin 30° = \Omega = 7.27 \times 10^{-5}\ s^{-1}$. The Ekman pumping at this latitude will be

$$w_s = -\frac{0.1 \times \pi}{1028 \times 7.27 \times 10^{-5} \times 3.33 \times 10^6}\sin\left(\frac{\pi}{2}\right) \approx -1.26 \times 10^{-6}\ \text{m s}^{-1}.$$

c) The relation between the Ekman pumping induced by the bottom friction (w_b) and the relative vorticity of the interior geostrophic flow (ξ_g) is

$$w_b = \frac{d}{2}\xi_g,$$

where d is the depth of the bottom Ekman layer. Because d is positive, ξ_g and w_b will have the same sign, which will also be that of w_s, if $w_b = w_s$. Since w_s is negative in our case, the relative vorticity ξ_g will be also negative (implying clockwise rotation).

d) Note that w_s is independent of x. The vertical volume flux through an arbitrary horizontal section of the ocean interior is

$$\begin{aligned} W &= \int_0^{L_x}\int_0^{L_y} w_s dy dx = L_x \int_0^{L_y} w_s dy = -\frac{\tau_0 L_x}{\rho_0 f}\int_0^{L_y} \frac{\pi}{L_y}\sin\left(\frac{\pi y}{L_y}\right) dy \\ &= \frac{\tau_0 L_x}{\rho_0 f}(\cos\pi - \cos 0) = \frac{-2 \times 0.1 \times 5 \times 10^6}{1028 \times 7.27 \times 10^{-5}} \approx -1.34 \times 10^7\ \text{m}^3\text{s}^{-1} = -13.4\ \text{Sv}. \end{aligned}$$

2. a) Kelvin waves propagate with the coast on their left in the Southern Hemisphere. Hence, the direction of propagation around New Zealand is counter-clockwise.
b) The wave speed is

$$c = \frac{L}{T} = \frac{3 \times 10^6 \text{ m}}{43200 \text{ s}} \approx 70 \text{ m s}^{-1}.$$

Knowing that the wave speed of a Kelvin wave is set by the ocean depth – as the speed of the shallow-water waves – according to the formula

$$c = \sqrt{gH}, \quad (1)$$

where H is the ocean depth, we can infer a depth

$$H = \frac{c^2}{g} = \frac{4900}{9.81} \approx 500 \text{ m}. \quad (2)$$

c) An estimate is given by the Rossby radius of deformation,

$$R = \frac{c}{|f|} = \frac{70 \ ms^{-1}}{10^{-4} \ s^{-1}} = 700 \text{ km}.$$

3. a) The maximum frequency of the internal waves that can be generated within the layer is $\omega_{max} = N$. The corresponding period is

$$T_{min} = \frac{2\pi}{\omega_{max}} = \frac{2\pi}{N} = \frac{2\pi}{3.5 \times 10^{-3} \text{ s}^{-1}} \approx 1.8 \times 10^3 \text{ s} = 30 \text{ min}.$$

b) The internal wave displaces seawater parcels along lines at an angle θ to the vertical given by the dispersion relation

$$\cos\theta = \frac{\omega}{N} = \frac{\frac{1}{2}N}{N} = \frac{1}{2}.$$

Hence, the angle is $\theta = \cos^{-1}(\frac{1}{2}) = 60°$.

4. a) The horizontal pressure gradient force balances the Coriolis force in the horizontal, whereas the vertical pressure gradient force balances the gravitational force in the vertical.

b) Since the current velocity vanishes at the bottom, the Coriolis force will be zero along the (horizontal) bottom surface. From the geostrophic balance, we obtain that the horizontal pressure gradient force must be zero, as well. Therefore, the bottom surface will be an isobaric surface.

c) From the equation of the geostrophic (near) surface meridional velocity,

$$v = \frac{g}{f}\frac{\partial \eta}{\partial x},$$

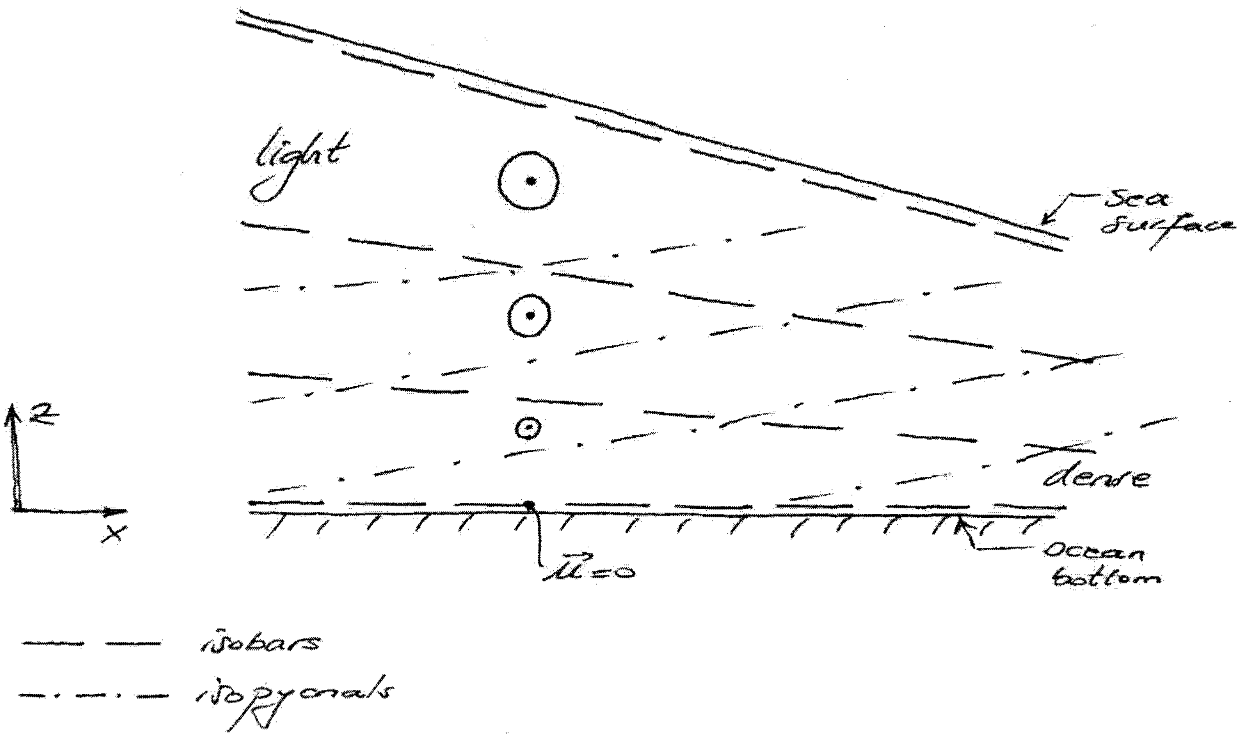

Figure 1: Sketch of a zonal section of a southward geostrophic current in the Northern Hemisphere.

we get

$$\frac{\partial \eta}{\partial x} = \frac{vf}{g} = \frac{-0.1 \times 10^{-4}}{9.81} \approx -10^{-6}.$$

d) See Fig. 1. Note that the sea surface is an isobaric surface, because the atmospheric pressure is assumed to be uniform there.