

Physics 4205/Amat 4180. Final exam. Winter 2003

(Closed Book and Notes)

1. **Boundary layer theory**

- (a) What does the boundary layer thickness  $\delta$  measure?
- (b) How does  $\delta$  depend on the Reynolds number ( $Re$ )?
- (c) If the boundary layer is thin what determines the horizontal (along the boundary) pressure gradient in the boundary layer?
- (d) If  $U$  is a typical velocity for  $u$ , in the  $x$ -direction (along the solid boundary) and  $L$  is a typical length scale along the boundary, what is the appropriate scaling for  $v$ , the velocity in the  $y$ -direction (perpendicular to the boundary) inside the boundary layer.

2. **Simple viscous flows**

- (a) *Couette flow*: In Couette flow, the upper plate is moving with velocity  $U$  while the lower plate is held stationary. The two plates are infinite in extent, parallel and separated by a distance  $h$  in the  $y$ -direction. What is the steady state solution for  $u(y)$ , the fluid velocity parallel to the plate?
- (b) Suppose the upper plane in (a) just started moving from the state of rest at  $t = 0$ . Explain the role of viscosity in establishing the steady state solution at large  $t$ ?
- (c) In Poiseuille flow, fluid is driven through a long cylindrical pipe by the pressure difference. Is the velocity profile linear or parabolic?
- (d) If you changed the pipes in your house to pipes of diameter two times bigger, what change in mass flow rate should you expect?

3. **Circulation**

- (a) Write down the potential, stream function and velocity for a 2D "irrotational" point vortex of strength  $\Gamma$ .
- (b) Where is vorticity concentrated in such a vortex?
- (c) If this vortex is placed in a viscous fluid of viscosity  $\nu$ , what is the approximate radius of the vortex after some time interval  $t$ ? Find the mean value of vorticity in the vortex at time  $t$ .

4. **Waves**

- (a) Describe the physical meaning of phase and group velocities.
- (b) Write down the phase and group velocities for internal waves in a stratified fluid of buoyancy frequency  $N$ .
- (c) What is the maximum frequency of internal wave? What is the direction of oscillation of fluid parcels in the wave when the frequency of oscillation is close to maximum frequency?
- (d) Shallow layer of fresh water overlies a deep layer of salt water. Is the phase speed of the barotropic wave faster or slower than that of the baroclinic wave? Show the phase speed of the baroclinic wave in this two-layer system.
- (e) Give the definition and explain the physical meaning of the Froude number and describe how the value of the Froude number changes in a flow going through a hydraulic jump.