Engineering 3821/Physics 3550
Final Exam
Tuesday, December 11, 2001

Answer all six (6) questions. Questions 1 and 2 are worth 5 marks. All other questions are worth 10 marks. The exam is out of 50. Write your answers on this exam paper. You may bring in one sheet (two sides) of notes. No other notes or books are permitted.

Tables of Laplace transforms are provided on the next page.

Time: 2.5 hours

Name:____

Student number:__________________________________

Check one: Engineering 3821 ______ Physics 3550 ______

TABLE 12.1

An Abbreviated List of Laplace Transform Pairs

TYPE	$f(t)(t > 0^-)$	$F(s)$
(inpulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

TABLE 12.2

An Abbreviated List of Operational Transforms

OPERATION	$f(t)$	$F(s)$
Multiplication by a constant	$Kf(t)$	$KF(s)$
Addition/subtraction	$f_1(t) + f_2(t) - f_3(t) + \cdots$	$F_1(s) + F_2(s) - F_3(s) + \cdots$
First derivative (time)	$\frac{df(t)}{dt}$	$sF(s) - f(0^-)$
Second derivative (time)	$\frac{d^2f(t)}{dt^2}$	$s^2F(s) - sf(0^-) - \frac{df(0^-)}{dt}$
nth derivative (time)	$\frac{d^nf(t)}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}\frac{df(0^-)}{dt} - s^{n-3}\frac{df^2(0^-)}{dt^2} - \cdots - \frac{d^{n-1}f(0^-)}{dt^{n-1}}$
Time integral	$\int_0^t f(x)\,dx$	$\frac{F(s)}{s}$
Translation in time	$f(t-a)u(t-a), a > 0$	$e^{-as}F(s)$
Translation in frequency	$e^{-at}f(t)$	$F(s+a)$
Scale changing	$f(at), a > 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
First derivative (s)	$tf(t)$	$-\frac{dF(s)}{ds}$
nth derivative (s)	$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
s integral	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$

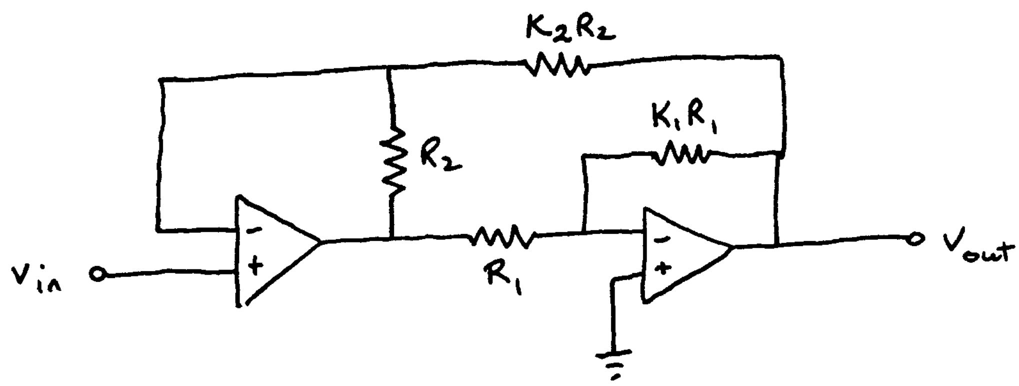
TABLE 12.3

Four Useful Transform Pairs

PAIR NUMBER	NATURE OF ROOTS	$F(S)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta} + \frac{K^*}{s+\alpha+j\beta}$	$2\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2} + \frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\lvert K\rvert e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $\lvert K\rvert\underline{/\theta}$.

1. [5 marks] Find the gain of the op-amp circuit shown in terms of the constants K_1 and K_2. Why should you avoid using this circuit with $K_1 = K_2$?

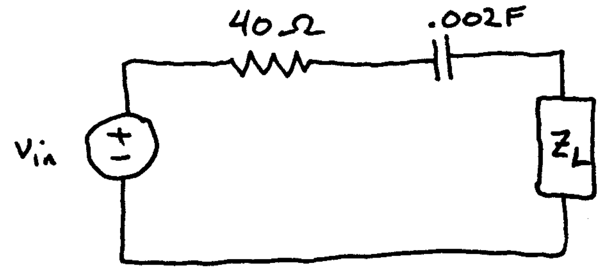


2. [5 marks] Design a filter circuit that has the transfer function $H(s)=\dfrac{s}{s+2\times10^{4}}$. Draw the circuit diagram. Give values for all circuit components used. Sketch the Bode diagram for this filter, and indicate the cutoff frequency on your diagram.

3. [10 marks]

a) In the circuit shown, the applied voltage is $220\cos 50t$ V (so 220 V is the amplitude, *not* the RMS voltage). The load impedance $Z_L = 20 + j30\ \Omega$. Calculate the following:

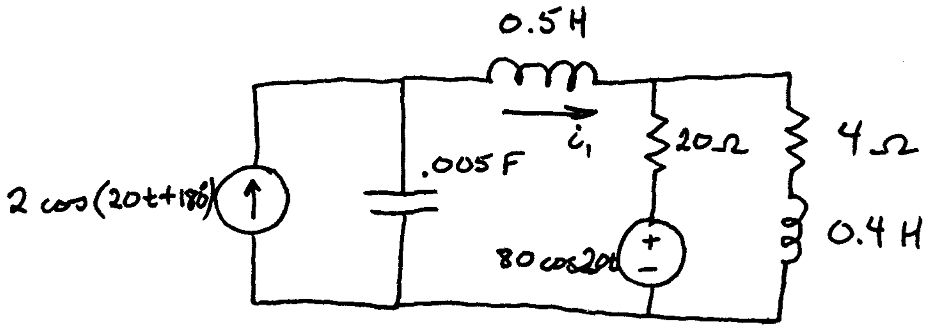
i. the current
ii. the average power dissipated *by the load*
iii. the reactive power absorbed *by the load*
iv. the power factor. Indicate whether the power factor is leading or lagging.



b) You are asked to change the load impedance Z_L so that the power transferred to the load is a maximum. Find the new values of

i. Z_L
ii. the current
iii. the average power dissipated *by the load*
iv. the reactive power absorbed *by the load*
v. the total reactive power absorbed *by the entire circuit.*
vi. the power factor.

4. [10 marks] Using complex impedances and phasors, find i_l in the circuit shown. Do not use Laplace transform methods.

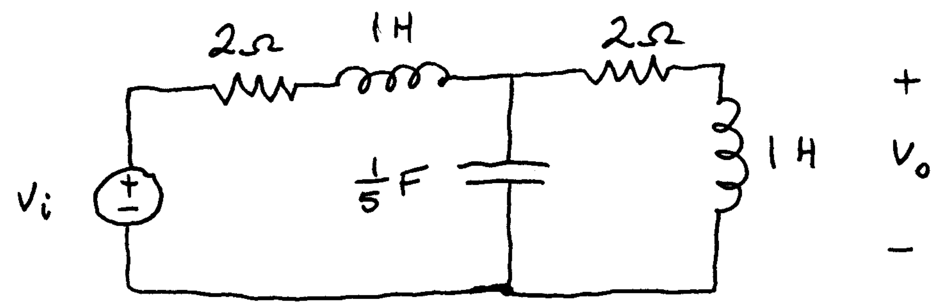


5. [10 marks]

a) Find the transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$ for the circuit shown.

b) Find $H(j\omega)$ for $\omega = 2\,\mathrm{s}^{-1}$.

c) Using your answer for (b), find the steady state response of the circuit if $v_i(t) = 4\cos(2t + 90^\circ)$.



6. [10 marks]

a) In the circuit shown, there is no energy stored in the inductor and capacitor when the switch is closed at $t = 0$. Use Laplace transform methods to find $v(t)$ for $t > 0$.

b) Based on your answer to (a), is the response of this circuit underdamped, overdamped, or critically damped?