Name:________________________

Student Number : _________________________

DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY
MEMORIAL UNIVERSITY OF NEWFOUNDLAND

Final Exam **Physics 3500** **December 5, 2007**
Fall 2007 **Time: 2 hours**

INSTRUCTIONS:

1. A complete paper consists of answers to **seven (7)** questions:
   four (4) questions from Part I (i.e. from among questions 1-5)**;**
   one (1) question from part II (i.e. either #6 or #7);
   one (1) question from part III (i.e. either #8 or #9); and
   one (1) question from part IV (i.e. either #10 or #11).

2. Marks are indicated next to each question. Questions in Part I are worth 10 marks each. Questions in Parts II, III, and IV are worth 20 marks each.

3. If you attempt more than the required number of questions in any part, your best answers will be used.

4. Write you answers in the space provided and use the backs of sheets as needed.

5. Be sure to **indicate direction** in some way for any quantities indicated as being vectors.

6. **Don't panic**.

POTENTIALLY USEFUL EQUATIONS CAN BE FOUND AT THE END OF THE PAPER.

For marking use only:

Part I					**Part II**		**Part III**		**Part IV**			
1	2	3	4	5	6	7	8	9	10	11		Total

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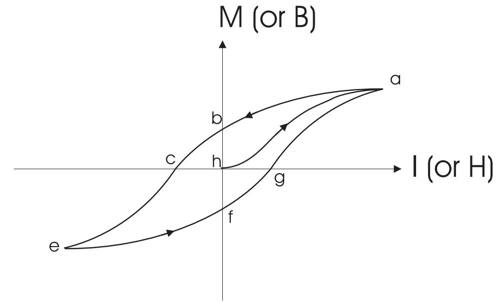
Part I: Complete four (4) questions from Part I (i.e. do four from questions 1-5).

[10] 1. A piece of material is placed in a long solenoid and a plot of its magnetization versus the current through the solenoid is found to be as shown.



(a) Is this material paramagnetic, ferromagnetic, or diamagnetic? Briefly justify your answer.

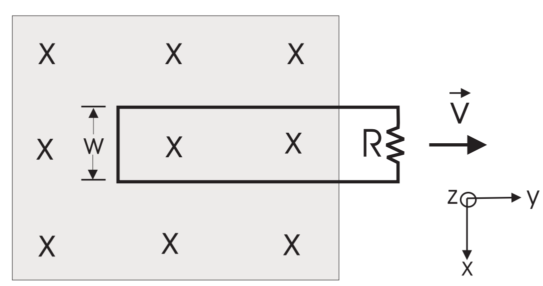
(b) Briefly describe what is meant by saturation and use the labels on the diagram to indicate portions of the curve for which the material is saturated.

(c) Which portions of the curve correspond to conditions in which the material is a permanent magnet?

[10] 2. A rectangular circuit of width w containing a resistance R is pulled from a region of uniform magnetic field, $\vec{B} = -B\,\hat{z}$, with a constant velocity $\vec{v} = v\,\hat{y}$ as shown.

(a) What current (magnitude and direction) flows in the circuit?

(b) What force is required to pull the circuit through the field at constant speed?



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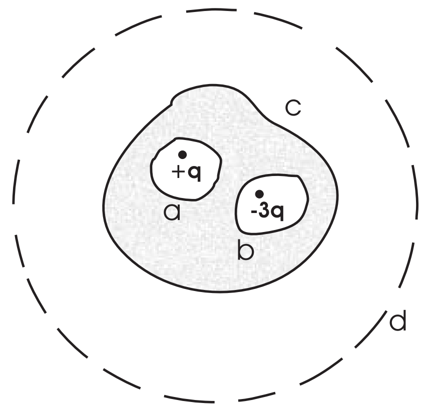
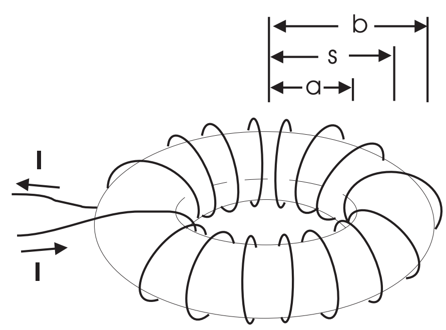
[10] 3. A conducting object contains two cavities labeled *a* and *b*. Cavity *a* contains a point charge $+q$ and cavity *b* contains a point charge $-3q$. There is no **net** charge on the conductor.

(a) What is the induced charge on the surfaces of the cavities (labeled *a* and *b*) and on the outer surface of the conductor (labeled *c*)?

(b) What is the total electric flux through the Gaussian surface labeled *d*?



[10] 4. A toroidal coil is wrapped around a doughnut-shaped form with an inner radius *a* and an outer radius *b*. There are 15 turns (i.e. the wire passes through the hole in the centre 15 times). Use Ampere's law to find the magnetic field inside the toroid (i.e. at a distance *s* from the centre where $a < s < b$).



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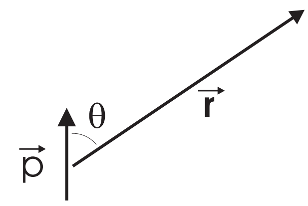
[10] 5. For an electric dipole at the origin and pointing in the z direction, the electric potential at a point (r, ϑ) in spherical coordinates is $V_{\text{dip}}(r, \vartheta) = \dfrac{\hat{r} \cdot \vec{p}}{4\pi\varepsilon_0 r^2}$. Find the electric field at the point (r, ϑ).



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Part II: Do one (1) question from part II (i.e. either #6 or #7).

[20] 6. A solid sphere of radius R is uniformly charged with a volume charge density ρ.

(a) First assume that the sphere is made of material with $\chi_e = 0$ (i.e. has no polarization).

(i) Find the electric field inside the sphere (i.e. at r from the centre where $r < R$).

(ii) Find the electric displacement $\vec{D}$ at r.

(b) Now assume that the same sphere is made of a **linear dielectric** with electric susceptibility $\chi_e = 3$.

(i) Find the electric field inside the sphere (i.e. at r from the centre where $r < R$). Hint: the distribution of free charge is the same as in part (a).

(ii) Find the polarization at r.

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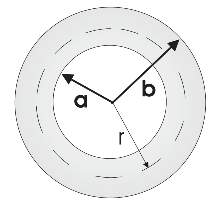
[20] 7. A spherical shell with an inner radius a and an outer radius b is made of a dielectric material with a permanent polarization $\vec{P}(\vec{r}) = kr\,\hat{r}$. There are **no free charges** on the object.



(a) Calculate the bound surface charge densities on the inner and outer surface and the bound volume charge density.

(b) Find the electric field within the dielectric (i.e. at r where $a < r < b$).
Hint: You can start from the electric displacement **or** use Gauss's law.

(c) Find the electric field outside of the sphere (i.e. at r where $r > b$).

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Part III: Do one (1) question from part III (i.e. either #8 or #9).

[20] 8. The magnetic vector potential in a region of space is $\vec{A} = k\,z^2\,\hat{x}$ in Cartesian coordinates.

(a) What magnetic field is produced by this magnetic vector potential?

(b) What is the current density $\vec{J}$ that gives rise to this magnetic field?

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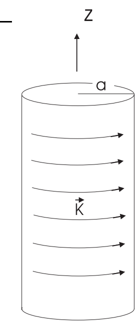
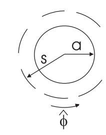
[20] 9. A time dependent current around a long solenoid oriented along the z axis can be modeled as a surface current $\vec{K} = k(t)\hat{\varphi}$ circulating around the surface of a cylinder of radius a as shown.

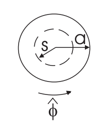


(a) Use Ampere's law to show that the magnetic field inside the cylinder is $\vec{B} = \mu_0 k(t)\hat{z}$. You may assume that the magnetic field outside the long solenoid is zero.

(b) What is the magnetic flux through a circular area bounded by the walls of the cylinder?



(c) Find an expression for the electric field outside of the cylinder (i.e. at s where $s > a$).



(d) Find an expression for the electric field inside the cylinder (i.e. at s where $s < a$).

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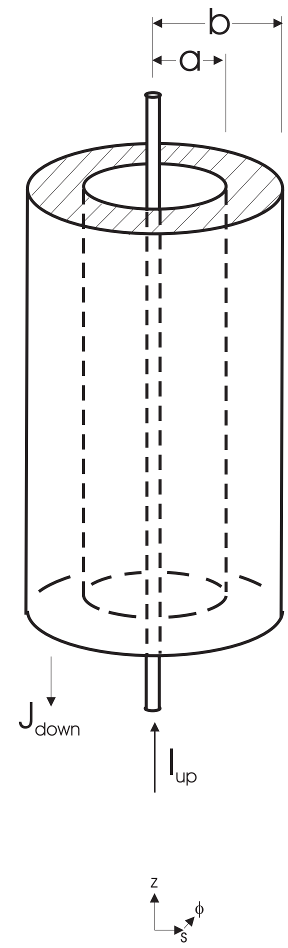
Part IV: Do one (1) question from part IV (i.e. either #10 or #11).



[20] 10. A thin wire carrying a current $\vec{I}_{\text{up}} = I_0\,\hat{z}$ is surrounded by a coaxial cylindrical shell with an inner radius *a* and an outer radius *b*. The current density in the cylindrical shell is uniform and given by

$$\vec{J}_{\text{down}} = -\frac{I_0}{\pi\left(b^2 - a^2\right)}\hat{z}\,.$$

(a) Find the magnetic field $\vec{B}$ in the space between the thin wire and the inner surface of the cylinder (i.e. for $s < a$).

(b) Show that the **total** current carried by the cylindrical shell between a and b is $\vec{I}_{\text{down}} = -I_0\,\hat{z}$.

(c) Find the magnetic field $\vec{B}$ outside of the cylindrical shell (i.e. for $s > b$). You may assume the result in (b) if you are unable to show it.

(d) Find the magnetic field within the wall of the cylindrical shell (i.e. at *s* where $a < s < b$).

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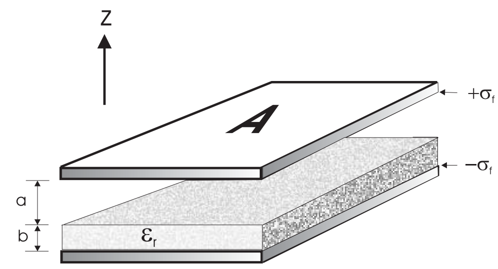
[20] 11. A parallel plate capacitor of area A is partially filled with a slab of **linear dielectric** having a relative permittivity ε_r and a thickness b. The remaining space between the dielectric and the upper conducting plate is empty and has thickness a. The free surface charge density on the upper conducting plate is $+\sigma_f$ and the free surface charge density on the lower conductor is $-\sigma_f$. Assume $A >> (a+b)$ so that the effect of the edges on the fields can be neglected.



(a) What is the electric displacement $\vec{D}$ between the conducting plates?

(b) What is the electric field in the dielectric?

(c) What is the electric field in the empty space between the dielectric and the upper conducting plate?

(d) Show that the capacitance of this system is $C = \dfrac{\varepsilon_0 \varepsilon_r A}{\varepsilon_r a + b}$.

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POTENTIALLY USEFUL INFORMATION:

1. Vector Calculus:

Gradient: $\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$ (Cartesian)

$$\vec{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\varphi}$$ (spherical)

Divergence: $\vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ (Cartesian)

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{r\sin\vartheta}\frac{\partial \sin\vartheta\, v_\vartheta}{\partial \vartheta} + \frac{1}{r\sin\vartheta}\frac{\partial v_\varphi}{\partial \phi}$$ (spherical)

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$ (cylindrical

Curl: $\vec{\nabla}\times\vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$ (Cartesian)

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\varphi}$$

(spherical)

$$\vec{\nabla}\times\vec{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\varphi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{z}$$ (cylindrical)

$\vec{\nabla}\cdot f\vec{A} = f(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot\vec{\nabla}f$, $\vec{\nabla}\times f\vec{A} = f(\vec{\nabla}\times\vec{A}) - \vec{A}\times\vec{\nabla}f$

$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$, $\nabla^2\vec{v} = \frac{\partial^2 v_x}{\partial x^2}\hat{x} + \frac{\partial^2 v_y}{\partial y^2}\hat{y} + \frac{\partial^2 v_z}{\partial z^2}\hat{z}$

$\int_{\vec{a}}^{\vec{b}} (\vec{\nabla}f)\cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$ Gradient Theorem

$\int_V (\vec{\nabla}\cdot\vec{v})\, d\tau = \oint_S \vec{v}\cdot d\vec{a}$ Divergence Theorem

$\int_S (\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \oint_P \vec{v}\cdot d\vec{l}$ Curl Theorem

$\int_V f(\vec{\nabla}\cdot\vec{A})\, d\tau = -\int_V (\vec{A}\cdot\vec{\nabla}f)\, d\tau + \oint_S f\vec{A}\cdot d\vec{a}$ Integration by parts

$d\tau = r^2 \sin\vartheta\, dr\, d\vartheta\, d\phi$ Volume element (spherical coordinates)

$\vec{\mathscr{r}} = \vec{\mathrm{r}} - \vec{\mathrm{r}}'$, $\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = -\nabla^2\left(\frac{1}{\mathscr{r}}\right) = 4\pi\delta^3(\vec{\mathscr{r}})$, $\int_{\text{all space}} f(\vec{r})\delta^3(\vec{r}-\vec{a})\, d\tau = f(\vec{a})$

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2. Electrostatics

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i$$ Coulomb's Law

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \quad , \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho(\vec{r})}{\varepsilon_0}$$ Gauss's Law

$$V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}}\vec{E}\cdot d\vec{l}\,, \qquad \vec{E} = -\vec{\nabla}V$$

$$V(\vec{b}) - V(\vec{a}) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$ (point charge)

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\vec{r}\,')}{\mathscr{r}}d\tau' \quad , \quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(\vec{r}\,')}{\mathscr{r}}da' \quad , \quad V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(\vec{r}\,')}{\mathscr{r}}dl'$$

$\nabla^2 V = 0$ (Laplace's equation) $\qquad \nabla^2 V = -\dfrac{\rho}{\varepsilon_0}$ (Poisson's equation)

$$W = \frac{1}{2}\sum_{i=1}^{n}q_i V(\vec{r}_i) \quad , \quad W = \frac{1}{2}\int\rho(\vec{r}\,')V(\vec{r}\,')d\tau' \quad , \quad W = \frac{\varepsilon_0}{2}\int_{\text{all space}}E^2\,d\tau$$

capacitors: $\quad C \equiv \dfrac{Q}{V} \quad , \quad W = \dfrac{1}{2}CV^2$

electric dipoles: $\vec{p} = q\vec{d} \quad , \quad V(\vec{r}) = \dfrac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}$

3. Electric Fields in Matter

$\vec{p} = \alpha\vec{E}$ (α is atomic polarizability) , $\quad \vec{N} = \vec{p}\times\vec{E}$ (torque on an electric dipole)

$\sigma_b = \vec{P}\cdot\hat{n}$ (bound surface charge density for polarized object)

$\rho_b = -\vec{\nabla}\cdot\vec{P}$ (bound volume charge density for a polarized object)

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\oint_S\frac{\sigma_b}{\mathscr{r}}da' + \frac{1}{4\pi\varepsilon_0}\int_V\frac{\rho_b}{\mathscr{r}}d\tau'$$ (electric potential due to a polarized object)

$\vec{D} \equiv \varepsilon_0\vec{E} + \vec{P}$ (electric displacement), $\quad \vec{\nabla}\cdot\vec{D} = \rho_f\,, \quad \oint\vec{D}\cdot d\vec{a} = Q_{f\,\text{enclosed}}$

$\vec{P} = \varepsilon_0\chi_e\vec{E}$ (polarization of linear dielectric material)

$\varepsilon = \varepsilon_0(1+\chi_e)$ (permittivity)

$\varepsilon_r \equiv \dfrac{\varepsilon}{\varepsilon_0} = 1 + \chi_e$ (dielectric constant or relative permittivity)

$$W = \frac{1}{2}\int\vec{D}\cdot\vec{E}\,d\tau$$

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4. Magnetostatics

$$\vec{F} = Q\left[\vec{E} + \left(\vec{v} \times \vec{B}\right)\right]$$

$$\vec{F}_{\text{mag}} = I\int d\vec{l} \times \vec{B}\,, \qquad \vec{F}_{\text{mag}} = \int \left(\vec{K} \times \vec{B}\right) da\,, \qquad \vec{F}_{\text{mag}} = \int \left(\vec{J} \times \vec{B}\right) d\tau$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$ (Continuity equation)

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2} dl' = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}\,' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$ (Biot-Savart law)

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$$ (magnetic field at a distance s from a long wire carrying current I)

$$f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$ (force between two long parallel wires)

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$ (Ampere's law, differential form)

$$\oint_{\substack{\text{amperian}\\ \text{loop}}} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$ (Ampere's law, differential form)

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}} d\tau'$$ (Magnetic Vector Potential)

$$\vec{B} = \vec{\nabla} \times \vec{A}\,, \qquad \vec{\nabla} \cdot \vec{A} = 0\,, \qquad \nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$\vec{m} = I\vec{a}$ (magnetic dipole moment), $\vec{N} = \vec{m} \times \vec{B}$ (torque on a magnetic dipole)

5. Magnetic Fields in Matter

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$ (bound volume current density for a magnetized object)

$$\vec{K}_b = \vec{M} \times \hat{n}$$ (bound surface current density for a magnetized object)

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\oint_S \frac{\vec{K}_b(\vec{r}\,')}{\mathscr{r}} da' + \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b(\vec{r}\,')}{\mathscr{r}} d\tau'$$ (mag. vect. pot. from a magnetized object)

$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$ (auxiliary field), $\vec{\nabla} \times \vec{H} = \vec{J}_f$, $\oint \vec{H} \cdot d\vec{l} = I_{f\,\text{enc}}$

$$\vec{M} = \chi_m \vec{H}$$ (magnetization of a linear medium)

$$\mu = \mu_0(1 + \chi_m)$$ (permeability of a linear medium)

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6. Electrodynamics

$\vec{J} = \sigma \vec{E}$ (Ohm's Law), $P = I^2 R$

$\Phi \equiv \int \vec{B} \cdot d\vec{a}$ (magnetic flux through a loop)

$\varepsilon = -\frac{d\Phi}{dt}$ (flux rule)

$\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$ (Faraday's law in integral form)

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (Faraday's law in differential form)

$M_{21} = \frac{\Phi_2}{I_1}$ (Mutual inductance of two loops), $L = \frac{\Phi}{I}$ (self-inductance)

$W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 \, d\tau$, $W = \frac{1}{2} \int_V \left(\vec{A} \cdot \vec{J} \right) d\tau$

7. Maxwell's equations

$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$ $\vec{\nabla} \cdot \vec{B} = 0$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

8. Some integrals we've used:

$$\int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin \vartheta \, dr \, d\vartheta \, d\varphi = \frac{4}{3} \pi R^3$$

$\int \frac{x dx}{\left(x^2 + a^2\right)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$ $\int \frac{dx}{\left(x^2 + a^2\right)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}}$

$\int \frac{dx}{x} = \ln x$ $\int \frac{dx}{a + bx} = \frac{1}{b} \ln(a + bx)$

$\int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}$ $\int \frac{x \, dx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$ $\int \frac{x \, dx}{\sqrt{a + bx}} = -\frac{2(2a - bx)}{3b^2} \sqrt{a + bx}$