PHYSICS 3500 Final Exam
December 9, 2005
2 pages, 2 hours, 100% total

You are required to complete 5 out of 6 questions from part 1 and both problems from part 2. Be complete but concise in your answers.

Part 1: Short-answer questions (Complete 5 of 6 – 10% each: 50% total)

1. The potential for an electric dipole with dipole moment $\vec{p}$ is
$$V_{dip} = \frac{\hat{r}\cdot\vec{p}}{4\pi\epsilon_o r^2}.$$
If the dipole is aligned with the z-axis, derive the equation for the electric field of this dipole as a function of the spherical coordinates r and θ. Use your equation to determine the electric field created by the dipole a distance r away on the positive x-axis.

2. The electric field near a large planar distribution of charge in the x-y plane is $\vec{E} = \frac{\sigma}{2\epsilon_o}\hat{z}$. Use this result to find the capacitance of a parallel-plate capacitor, where the plates have area A, are separated by distance d and you can assume the plates are large.

3. Starting with Gauss's law in differential form, $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_o}$, derive Gauss's law in integral form. Be sure to be specific and clear about what steps you make and why you are making them.

4. Most materials are either paramagnetic or diamagnetic. Explain what this means in terms of their response to an external magnetic field. Briefly give the physical reason for each type of response and explain why some materials exhibit paramagnetism and some diamagnetism.

5. A dedicated instructor drops a bar magnet through a copper pipe while he drops a non-magnetized piece of metal through a second copper pipe. The magnet takes much longer to fall through the pipe than the non-magnetized piece. Explain clearly why this occurs.

6. List the four types of current distribution symmetry for which Amperian loops may be used to solve Ampere's law problems. For each, give the direction of the magnetic field and draw a diagram showing the orientation and shape of the Amperian loop that allows one to solve the problem.

Part 2: Problems (Complete both -- 50% total)

7. A rubber ball of radius a is centered on the origin. It has permittivity ϵ and it is also electrically charged so that it has a free charge density $\rho = Cr$, where C is a constant, throughout its volume.

   a.) Find the electric displacement as a function of r using Gauss's law in matter for:

   7 i.) $r<a$ and

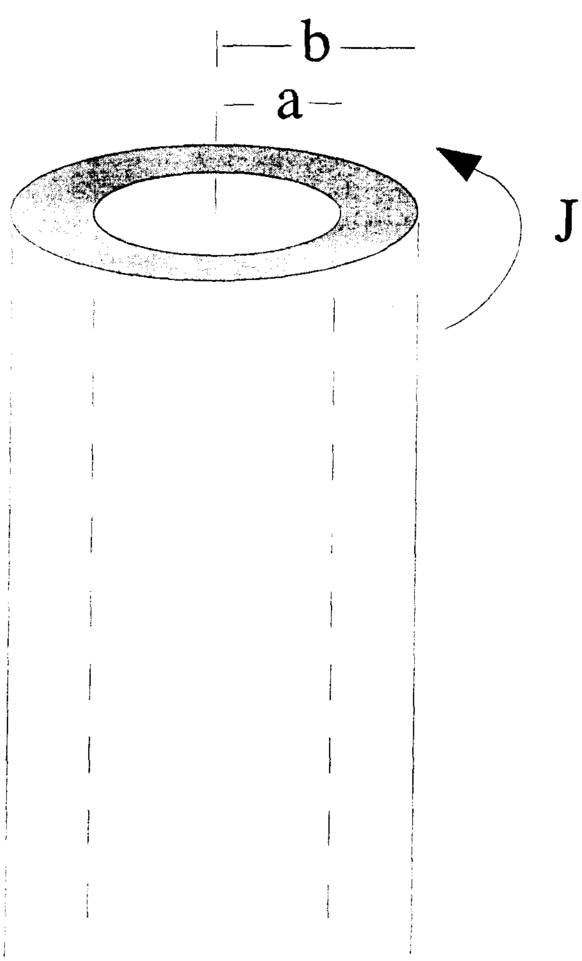
   7 ii.) $r>a$

   b.) Using your answer for the electric displacement from part a.), find the electric field as a function of r for:

   3 i.) $r<a$ and

   3 ii.) $r>a$

8. A long conducting cylindrical shell with inner radius a and outer radius b has a uniform volume current density $\vec{J} = J\hat{\phi}$ flowing in the circumferential direction. Use s as the radial distance from the shell's central axis.



   a.) Using Ampere's law in integral form find the magnetic field $\vec{B}$

   8 i.) Interior to the shell ($s<a$)

   8 ii.) Inside the shell ($a<s<b$)

   14. b.) Find the total energy stored in the magnetic field of a length l of this shell.