Name:________________________

DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY
MEMORIAL UNIVERSITY OF NEWFOUNDLAND

Final Exam	**Physics 3410**	**April 15, 2006**
Winter 2006		**Time: 2 hours**

INSTRUCTIONS:
1. Attempt **ALL** questions (1-6) in **Part 1**. Attempt **1** out of the **2** questions (7,8) in **Part 2.** Marks assigned to each question are indicated in the margin.
2. Write you answers in the space provided and use the backs of sheets as needed. An extra blank page is provided at the end for Part 2 if needed.
3. Use ink. Do not erase or use whiteout. Indicate deletion by a line drawn neatly through unwanted material.
4. If what anything is not clear, ask. Don't panic.

POTENTIALLY USEFUL EQUATIONS:

$\wp(s) = \dfrac{e^{-E(s)/kT}}{Z}$ Boltzmann Distribution (Canonical Distribution)

$\wp(s) = \dfrac{e^{-[E(s)-\mu N(s)]/kT}}{\mathcal{Z}}$ Grand Canonical Distribution

$Z = \sum_s e^{-\beta E(s)}$ Partition function

$\mathcal{Z} = \sum_s e^{-\beta[E(s)-\mu N(s)]}$ Grand Partition Function

$Z_1 = \sum_s e^{-E(s)/kT}$ Single particle partition function

$\overline{E} = -\dfrac{1}{Z}\dfrac{\partial Z}{\partial \beta}$ $\qquad$ $l_Q = \dfrac{h}{\sqrt{2\pi mkT}}$

power per unit area $= \sigma T^4$

$\sum_{n=0}^{\infty} x^n = (1-x)^{-1}$ for $x<1$. Infinite geometric series

$N_A = 6.02\times10^{23}$ $\qquad$ $h = 6.626\times10^{-34}\ \text{J}\cdot\text{s} = 4.136\times10^{-15}\ \text{eV}\cdot\text{s}$

$k = 1.381\times10^{-23}\ \text{J/K} = 8.617\times10^{-5}\ \text{eV/K}$ $\qquad$ $\beta = (kT)^{-1}$

$e^x = 1 + x + \dfrac{x^2}{2} + \cdots$ $\qquad$ $\cosh(x) = \dfrac{1}{2}\left(e^x + e^{-x}\right)$

$(1+x)^{-1} \approx 1 - x + \cdots$ for $|x| << 1$ $\qquad$ $\sinh(x) = \dfrac{1}{2}\left(e^x - e^{-x}\right)$

$\ln(1+x) \approx x - \dfrac{x^2}{2} + \cdots$ for $|x| << 1$ $\qquad$ $\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$

$N! \approx N^N e^{-N}\sqrt{2\pi N}$

$\ln N! \approx N\ln N - N$

1	2	3	4	5	6	7	8	Total

Name:__________________________

PART 1: Do questions 1, 2, 3, 4, 5 and 6

[10] 1. (a) What is meant by the *fundamental assumption of statistical mechanics*?

(b) Consider a small system in thermal equilibrium with a large reservoir. Comment briefly on the relationship between the probability that the small system is in a particular microstate and the multiplicity of the reservoir.

(c) The nucleus of the ^{14}N atom is a spin-1 particle. The allowed values of the z component of the ^{14}N nucleus are $\mu_z = -\mu_{14N}$, $\mu_z = 0$, and $\mu_z = +\mu_{14N}$ where $\mu_{14N} = 2.04\times10^{-27}$ J/T. The energy of a magnetic moment in a field of magnitude B is $E = -\mu_z B$.

(i) Find an expression for the partition function of a ^{14}N nucleus in a magnetic field of magnitude B and evaluate it for $T = 298\text{ K}$ and $B = 9.4\text{ T}$.

(ii) Calculate the average magnetic moment, $\overline{\mu}_z$, for ^{14}N nuclei at $T = 298\text{ K}$ in a magnetic field of magnitude $B = 9.4\text{ T}$

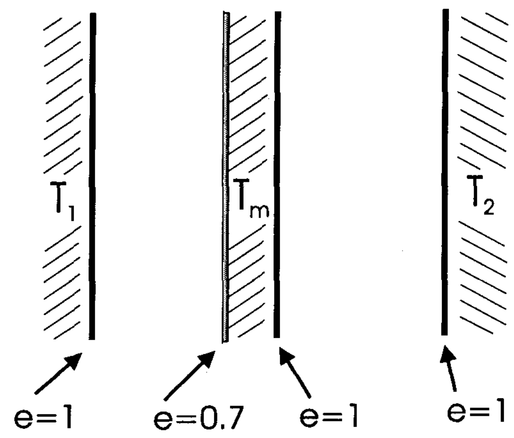
Name:________________________

[5] 2. (a) Briefly discuss how the characteristic length scale for fluctuations in a parameter like density or magnetization might change as a system approaches a critical point. Comment on how this observation suggests that the critical point can be identified with the fixed point in a recursion relation for coupling obtained from a renormalization group calculation.

(b) The mean field model predicts that below the Curie temperature, the temperature dependence of the magnetization goes like $M \propto (T_C - T)^{1/2}$. Experimentally, the behaviour of 3-dimensional ferromagnets is found to be closer to $M \propto (T_C - T)^{1/3}$. The difference is presumably because the mean field approximation neglects some elements of the behaviour of real systems near a critical point. Identify one such behaviour that is neglected by the mean field approximation.

Name:________________________

[5] 3. Two large parallel surfaces, 1 and 2, are at temperatures $T_1 = 298\text{ K}$ and $T_2 = 77\text{ K}$ respectively. Both surfaces have emissivity $e = 1.0$. A thin metal sheet is inserted into the space between these surfaces. The emissivity of the surface of this sheet facing surface 1 is $e = 0.70$. The emissivity of the surface of this sheet facing surface 2 is $e = 1.0$. What is the equilibrium temperature, T_m, of the middle sheet?



Name:________________________

[5] 4. The Debye density of phonon states has the form $g_{\text{Debye}}(\varepsilon) = \frac{12\pi V}{h^3 c_s^3}\varepsilon^2$ for $\varepsilon < kT_D$ and $g_{\text{Debye}}(\varepsilon) = 0$ for $\varepsilon > kT_D$. The Debye model approximates the observed heat capacity of a solid well for $T << T_D$ and for $T >> T_D$.

(a) What aspect(s) of the Debye model account(s) for the agreement with observations at low temperature? Briefly comment on why this is a good approximation for low temperature behaviour.

(b) What aspect(s) of the Debye model account(s) for the agreement with observations at high temperature?

Name:___________________________

[10] 5. (a) The Planck Distribution, $\bar{n}_{\mathrm{Pl}} = \frac{1}{e^{\varepsilon/kT} - 1}$, has the same form as the Bose-Einstein distribution would have for particles with chemical potential $\mu = 0$. To what kinds of particles is the Planck Distribution applied and why might $\mu = 0$ be reasonable for such particles.

(b) The allowed wavelengths for photon modes in a $L \times L \times L$ box are $\lambda = \frac{2L}{\sqrt{n_x^2 + n_y^2 + n_z^2}}$ where n_x, n_y, and n_z are positive integers. The energy of a photon of wavelength λ is $\varepsilon = \frac{hc}{\lambda}$. Show that the photon density of states is given by $g(\varepsilon) = \frac{8\pi L^3}{h^3 c^3}\varepsilon^2$.

Name:__________________________

[10] 6. (a) For an Ising model with no external field, the internal energy in a domain is $U = -\varepsilon \sum_{\substack{\text{nearest} \\ \text{neighbours}}} s_i s_j$ where $s_i = \pm 1$ denotes whether dipoles are parallel or antiparallel to the z axis. Briefly describe what is meant by a mean field approximation and show that such an approximation yields $\bar{s} = \tanh(\beta \varepsilon n \bar{s})$ where n is the number of nearest-neighbours.

(b) Using $\lim_{x \to 0} \tanh(x) = x$ and the result in part (b), find an expression for the temperature, T_C, at which the system first displays a spontaneous non-zero magnetization upon cooling. It may be helpful to sketch a graph that represents your solution to the transcendental equation for $\bar{s}$.

Name:___________________________

PART 2: Do **one (1)** out of the **two** questions (7 or 8) in Part 2. (an extra blank page is available at the end if needed)

[15] 7. (a) Consider a single particle boson state $(\text{energy} = \varepsilon)$ that can be occupied by an integer number of bosons. Derive the grand partition function for this state by treating the state as a "system" and the other single particle states as a "reservoir" of particles with chemical potential μ.

(b) Use your result from (a) to derive the Bose-Einstein distribution function $\bar{n}_{BE} = \frac{1}{e^{(\varepsilon-\mu)/kT} - 1}$ from the definition $\bar{n} = \sum_n n\wp(n)$ for the average number of particles in a particular single particle state. Hint: it may be helpful to note $ne^{-nx} = -\frac{\partial e^{-nx}}{\partial x}$.

(c) Show that for temperatures low enough to give a macroscopic occupation of the lowest single particle state (with energy ε_0), the number of particles in the ground state is $N_0 = \frac{kT}{\varepsilon_0 - \mu}$. What does this imply regarding the low temperature limit of the chemical potential for bosons.

(d) Sketch the temperature dependence of the chemical potential for a gas of weakly interacting bosons. Indicate the Bose-Einstein condensation temperature on your graph.

Name:________________________

[15] 8. Consider a gas of N spin-1/2 fermions confined to a one dimensional space of length L so that the allowed single particle state energies are $\varepsilon = \frac{h^2 n_x^2}{8mL^2}$.

(a) Derive an expression for the density of states, $g(\varepsilon)$ for this system of particles.

(b) Based on your result for part (b), do you expect the chemical potential to increase, decrease, or stay constant as temperature is raised slightly from $T = 0\text{ K}$? Briefly justify your answer.

(c) The chemical potential must satisfy $N = \int_0^\infty g(\varepsilon)\bar{n}_{\text{FD}} d\varepsilon$ where

$\bar{n}_{\text{FD}} = \frac{1}{e^{(\varepsilon-\mu)/kT}+1}$. Obtain an expression for the chemical potential at $T >> \varepsilon_F / k$ in terms of N, L, kT, and the quantum length l_Q. You can assume that the chemical potential is large and negative for $T >> \varepsilon_F / k$ and you may find it useful to note that $\int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx = \sqrt{\pi}$.

Name:_________________________

Extra page for part 2: