PHYS 3300 - Final Examination
December 8, 2005
Instructor: Daniel Bourgault

1. (a) Write down the system of equations that represent an inertial balance? 2 pt.
   (b) What are the characteristics of an ocean flow in inertial balance? 3 pt.

2. (a) Write down the system of equations that represent an Ekman balance? 2 pt.
   (b) What boundary condition at the sea surface was imposed by Ekman to solve these equations? 1 pt.
   (c) What are the characteristics of the solution of this system of equations? Support your answer with a clear sketch. 4 pt.
   (d) Explain what the Ekman transport is and how it relates to coastal upwelling. Support your answer with a sketch. 2 pt.

3. Show that for a homogeneous ocean (i.e. constant density ρ_0) in hydrostatic equilibrium, the geostrophic balance 4 pt.

$$fv = \frac{1}{\rho_0}\frac{\partial p}{\partial x},$$

can be expressed as

$$fv = g\frac{\partial \eta}{\partial x},$$

where η is the sea-surface elevation (other variables follow the standard notation introduced in class).

4. Using the information provided in Figure 1, estimate as best as you can the geostrophic current (intensity and direction) at the location of the $\times$ sign in the North Atlantic. 4 pt.

5. Figure 2 shows a 7 days prediction of the water level in St. John's. Explain qualitatively (i.e. no equations needed) the features of this tidal signal. In particular explain:
   (a) Why are there two tides per day while the Earth with respect to the Moon and the Sun makes only one revolution per day; 2 pt.
   (b) Why two consecutive tides have quite different amplitudes; 2 pt.
   (c) Why the average tidal range (i.e. the difference between high and low tide) decreases with time over the prediction period. 2 pt.

   Support your answer with a clear sketch of the Earth-Moon-Sun system and show with vectors the forces involved.

6. Assume that the density profile in some region of the ocean can be represented by the relation

$$\rho(z) = \rho_0 - \gamma z,$$

where $\rho_0 = 1023\ \mathrm{kg\,m^{-3}}$ and $\gamma = 0.01\ \mathrm{kg\,m^{-4}}$.

   (a) What is the buoyancy frequency at $z = -100$ m? Show your work. 2 pt.

   (b) What is the pressure at $z = -100$ m? Show your work. 2 pt.

7. Figure 3 shows an internal wavetrain propagating at the interface of a two layer system in the St. Lawrence Estuary. The densities of the top ~~layer~~ and bottom layers are, respectively, $\rho_1 = 1020\ \mathrm{kg\,m^{-3}}$ and $\rho_2 = 1025\ \mathrm{kg\,m^{-3}}$.

   (a) Given the information on the figure would you consider the internal waves seen between 900 m and 1100 m from the shore to be "shallow water" or "deep water" waves? Justify your answer. 2 pt.

   (b) Estimate the phase speed of the wave at 1000 m from the shore. 2 pt.

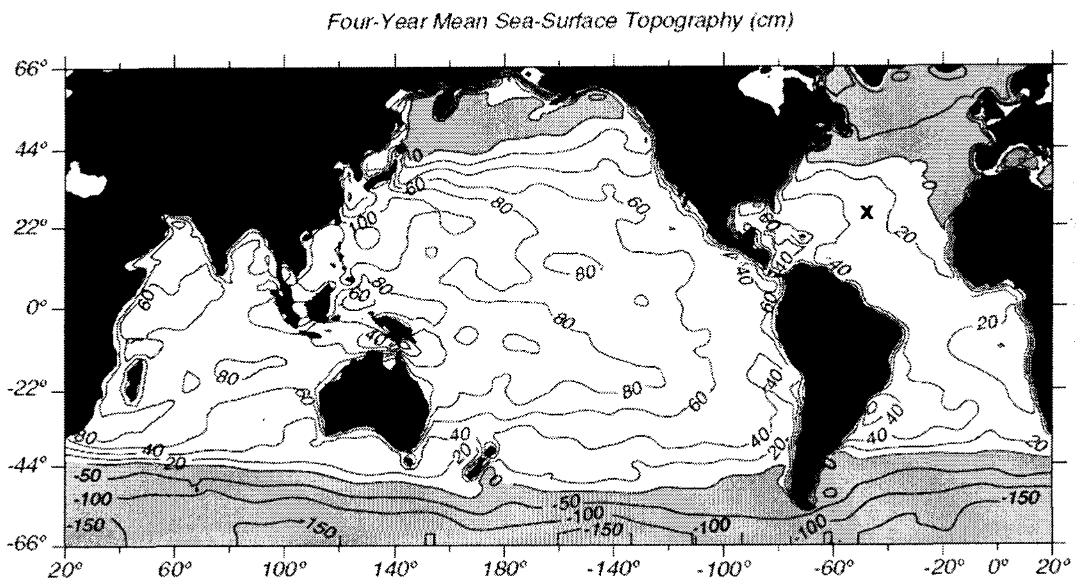

Figure 1: Global distribution of the sea surface height.

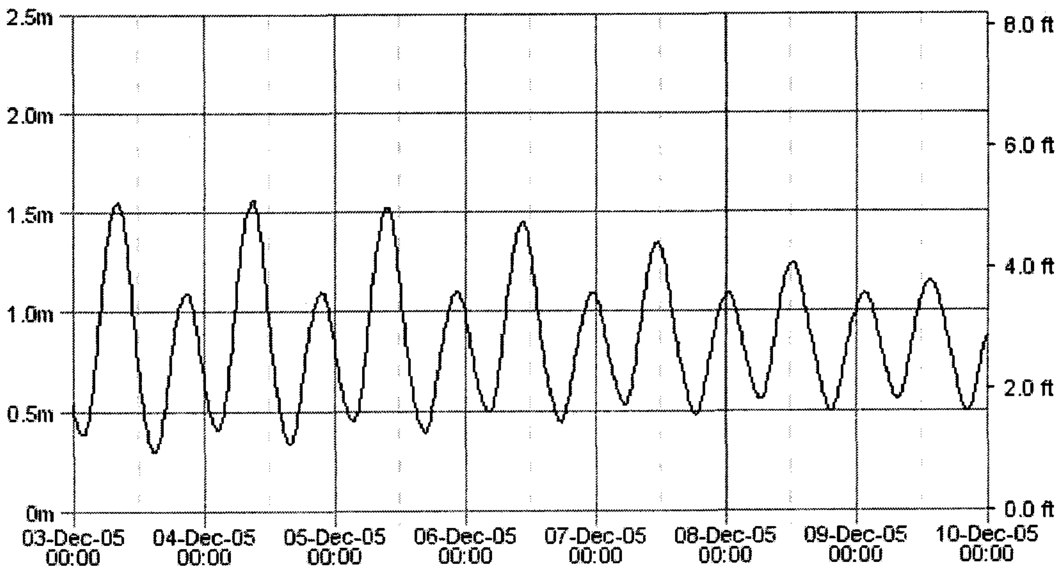

Figure 2: 7 days prediction of the water level in St. John's.

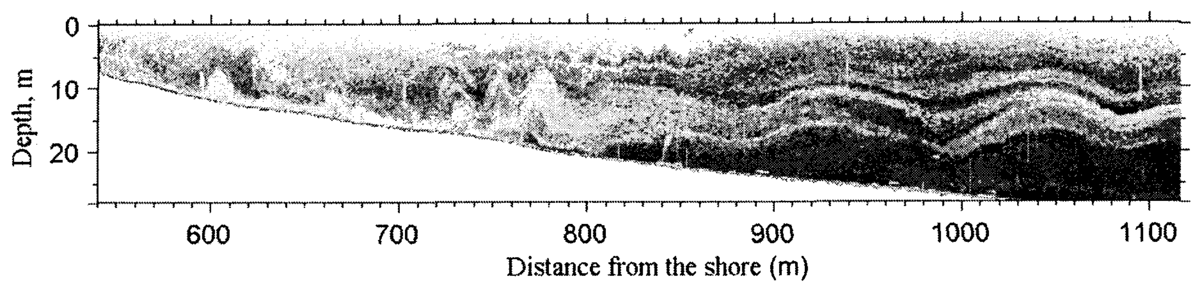

Figure 3: An internal wavetrain running into a sloping boundary in the St. Lawrence Estuary. The waves propagate from right to left. These observations were collected in 2004 by Marina for her Ph.D. research.

Equation Sheet

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + A_x\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + A_z\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + A_x\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + A_z\frac{\partial^2 v}{\partial z^2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$f = 2\Omega\sin\phi$$

$$\Omega = \frac{2\pi}{1\text{ day}} = \frac{2\pi}{86400\text{ s}} = 7.2722\times10^{-5}\,\text{rad}\,\text{s}^{-1}$$

$$N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z},\quad g' = g\frac{\Delta\rho}{\rho_0}$$

$$c = \sqrt{\frac{g}{k}\tanh(kH)},\quad c = \sqrt{g'H_e},\quad H_e = \frac{h_1 h_2}{h_1 + h_2},\quad c = \sqrt{\frac{g}{k}\frac{\Delta\rho}{(\rho_2+\rho_1)}}$$

$$w_E = \frac{1}{\rho_0 f}\left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y}\right)$$

$$r = V/|f|,\quad T = 2\pi/|f|$$

$$R = \sqrt{gH}/|f|$$

$$D_E = \pi\sqrt{\frac{2A_z}{|f|}}$$

$$g = 9.81\,\text{m}\,\text{s}^{-2}$$

$$1^\circ \text{ of latitude} = 111\text{ km}$$