PHYS 3300 - Final Examination
Saturday, December 11, 2004
Instructor: Daniel Bourgault

1. (a) Show that for a homogeneous ocean (i.e. a one-layer ocean where ρ = constant), in hydrostatic equilibrium, the geostrophic equation 10

$$fv = \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (1)$$

can be expressed as

$$fv = g\frac{\partial \eta}{\partial x} \qquad (2)$$

where η is the position of the free surface (other variables follow standard notation).

(b) What is the advantage of using (2) instead of (1) for understanding ocean circulation? Answer with one or two sentences. 5

(c) Write down the equations for a *two-layer* geostrophic ocean. Take the position of the free surface and interface as η_1 and η_2, respectively. Define any other new variables introduced. (As in (a) consider only the x-component of the equations i.e. $u_1 = u_2 = 0$). 5

(d) What must the slope of the interface be in (c) as a function of the slope of the free surface such that the bottom layer is motionless? Express your results in terms of the densities of the top and bottom layers, which are considered known. 5

2. (a) Write down the system of equations that describes inertial motions. 5

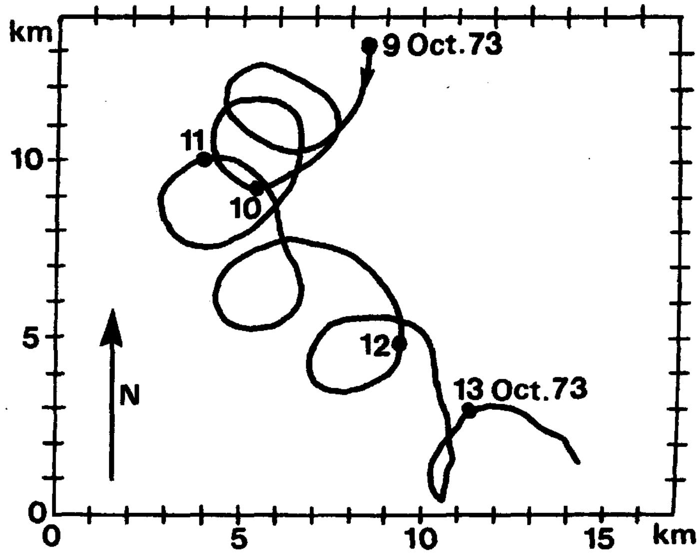
(b) Recalling that the solution to the set of equations in (b) is (standard notation) 5

$$u = V_0 \sin(ft), \; v = V_0 \cos(ft), \qquad (3)$$

where V_0 is a constant, prove that water parcels subject to inertial motions follow circular paths of radius $V_0/|f|$.

(c) Figure 1 shows the trajectory of a water parcel observed at latitude 47°09′ N. Using the marks counting the days along the curve, show that this set of observations reveals the presence of inertial oscillations. 5

Figure 1: Inertical circles?



3. (a) Ekman theory assumes a balance between frictional and Coriolis forces. This balance can be justified if the Ekman number 5

$$E_K \equiv \frac{A_V}{f d^2} \tag{4}$$

is close to 1, where d is a depth scale (other variables follow standard notation). Derive this Ekman number E_K by carrying out a scaling analysis.

(b) Explain coastal upwelling and what causes it? Support your answer with a clear diagram. 5

4. (a) Write down the system of equations used by Stommel to explain the circulation of the North Atlantic. 5

(b) What is added in these equations as compared to Sverdrup's analysis? 5

(c) i) What type of forcing was applied to these equations (you can answer either with an equation or with a clear diagram.) and ii) what does this forcing represent in term of the real Nature of the North Atlantic environment? 5

(d) Stommel examined three different situations. Identify them and make one clear diagram of the circulation pattern for each of these three situations. 5

(e) If the Westerlies and the Trade winds in the Northern Hemisphere changed direction, how would that affect the sea surface height and the circulation in the North Atlantic? Support your answer using vorticity arguments. 5

BONUS: Tie a bowline knot with the piece of rope provided with this exam. 5

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + A_H\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + A_V\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + A_H\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + A_V\frac{\partial^2 v}{\partial z^2}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = K_H\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + K_V\frac{\partial^2 T}{\partial z^2}$$

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} + v\frac{\partial S}{\partial y} + w\frac{\partial S}{\partial z} = K_H\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}\right) + K_V\frac{\partial^2 S}{\partial z^2}$$

$$\rho = f(S, T, p)$$

$$\frac{f + \zeta}{H} = \text{constant}, \quad \zeta = \partial v/\partial x - \partial u/\partial y$$

$$f = 2\Omega \sin\phi$$

$$g = 9.81\,\mathrm{m\,s^{-2}}$$