MEMORIAL UNIVERSITY OF NEWFOUNDLAND
Department of Physics and Physical Oceanography
Physics 3230 (Classical Mechanics II)
Final Examination
Friday April 13, 2007, 3:00-5:00 p.m.

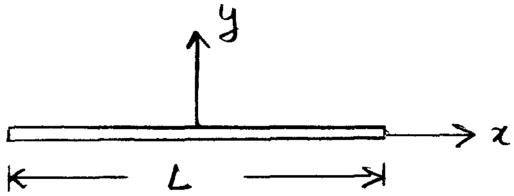
Answer all 5 questions

Formulae

$$
\begin{aligned}
r &= \frac{c}{1+\varepsilon\cos\phi} \\
c &= \frac{l^2}{\gamma\mu} \\
\gamma &= GmM \quad \mu = \frac{mM}{m+M} \\
g &= \frac{GM}{R^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} &= \mathbf{v}_{O'} + \mathbf{v}' + \boldsymbol{\omega}\times\mathbf{r}' \\
\mathbf{a} &= \mathbf{a}_{O'} + \mathbf{a}' + 2\boldsymbol{\omega}\times\mathbf{v}' + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}') + \boldsymbol{\alpha}\times\mathbf{r}'
\end{aligned}
$$

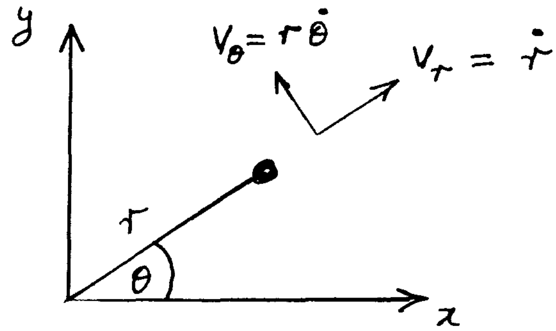
$$
\begin{aligned}
\{H\} &= [I]\{\omega\} \\
p_k &= \frac{\partial L}{\partial \dot{q}_k} \\
H &= \sum_{i=1}^{n} p_i\dot{q}_i - L \\
\dot{q}_k &= \frac{\partial H}{\partial p_k} \quad (k=1,\ldots n) \\
\dot{p}_k &= -\frac{\partial H}{\partial q_k} \quad (k=1,\ldots,n)
\end{aligned}
$$



Rod of mass M, length L

$I_{xx} = 0 \quad I_{yy} = I_{zz} = \frac{1}{12}ML^2$

$I_{xy} = I_{yz} = I_{zx} = 0$



Qu. 1.

(a) **[10 marks]** A spacecraft is describing an elliptic orbit around the earth. The orbit has minimum radius (measured from the earth's centre) r_A at point A and maximum radius r_B at point B. Assuming that the mass m of the spacecraft is much smaller than the mass M of the earth, show that

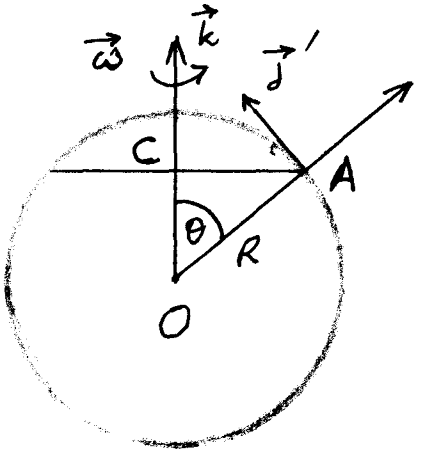
$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR^2}{h^2}$$

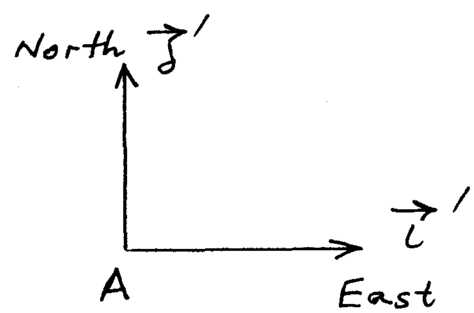
where g is the acceleration due to gravity at the earth's surface, R is the radius of the earth and h is the angular momentum of the spacecraft per unit mass, i.e. $h = \ell/m$ where $\ell =$ angular momentum of spacecraft about earth's centre.

[b] **[10 marks]** The minimum altitude of the spacecraft above the earth's surface is $h_A = 2640\ km$ and the maximum altitude is $h_B = 10560\ km$. The earth's radius is $R = 6370\ km$. Determine the speed of the spacecraft at A and B.

Qu. 2 [20 marks]

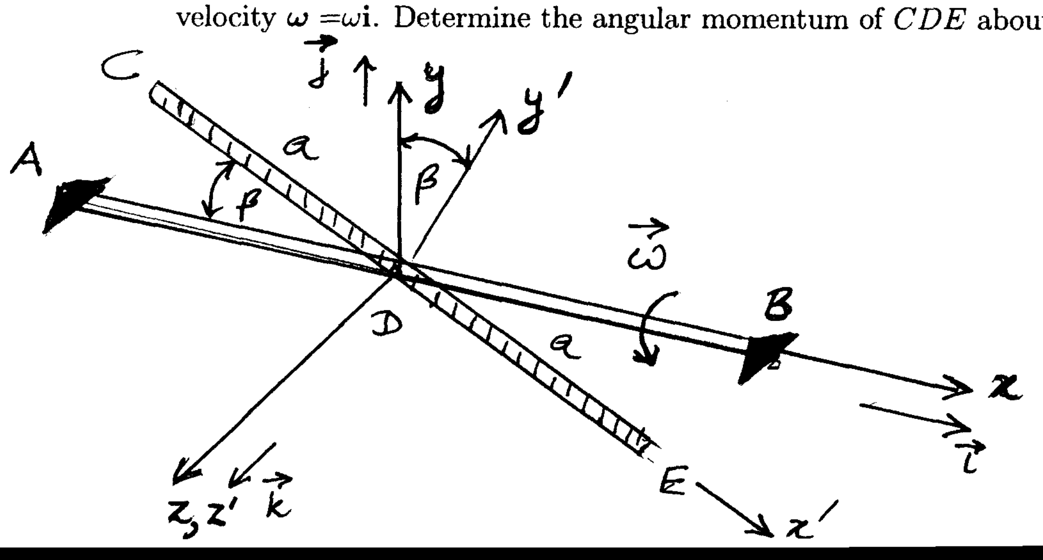
An object of mass $m = 1500\ kg$ is observed to be at a height of $5000\ km$ vertically above a point A on the earth's surface at colatitude $\theta = 30^0$. Relative to A, the object has a constant velocity of $8000\ m/s$ due south (direction $-\vec{j}\,'$). Determine the resultant force on the object in terms of unit vectors at A : $\vec{i}\,'$poining east, $\vec{j}\,'$pointing north and $\vec{k}\,'$ pointing vertically upwards. The earth's angular speed is $\omega = 7.3 \times 10^{-5}\ rad/s$ and its radius is $R = 6370\ km$.

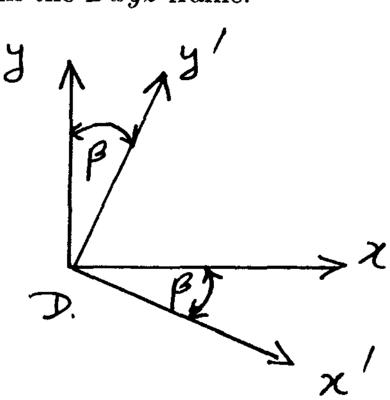




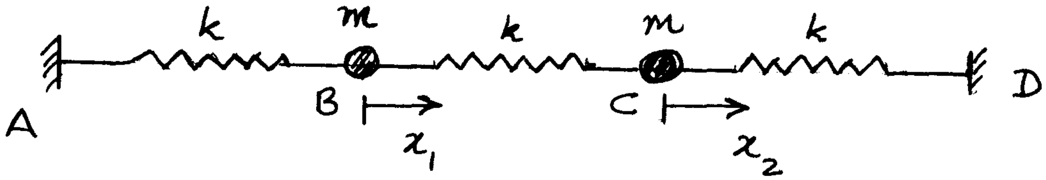
Qu. 3 [20 marks]

Two uniform rods CD and DE each of mass m and length a are welded to shaft AB as shown to form a straight rod CDE of mass $2m$ and length $2a$. At the instant shown, both CDE and AB lie in the $x - y$ plane and the structure rotates around AB with angular velocity $\omega = \omega$i. Determine the angular momentum of CDE about D in the $Dxyz$ frame.





Qu. 4 [20 marks]
AB, BC and CD are identical springs of negligible mass and stiffness k. The masses m fixed to the springs at B and C are displaced by small distances x_1 and x_2 from their equilibrium positions along the line of the springs. Show that the system has natural frequencies $\omega_1 = \sqrt{k/m}$ and $\omega_2 = \sqrt{3k/m}$ and find the normal modes. It is not necessary to find the normal coordinates.



Qu. 5[20 marks]
Two particles A and B of masses m and M respectively are connected by a light inextensible string of length a which passes through a smooth hole O in a smooth horizontal table.Particle B is suspended below the table and particle A rests on the table. Particle A has two degrees of freedom r, θ in the plane of the table. Using generalised coordinates $q_1 = r$ and $q_2 = \theta$ as shown, derive Hamilton's equations of motion and show that the radial acceleration of A is inversely proportional to the cube of its distance from O.