PHYSICS 3220. Final Examination. Fall 2007. Time 2 hours.

Solve all problems. Show all details of your physical and mathematical arguments. Two pages, written on both sides, of any aides to memory are allowed.

1. Consider an object that is thrown vertically up with an initial speed v_0 in a medium with linear resistance. **(a)** Measuring y *upward* from the point of release, derive an expression for the object's velocity as a function of time v (t). **(b)** Using the vdv/dy rule derive a relationship between the object's velocity v and its position y. **(c)** Find the highest point y_{max}.

   Hint: use the integrals $\int \frac{dx}{a+x} = \ln(a+x)$ and $\int \frac{xdx}{a+x} = a + x - a\ln(a+x)$

2. The first couple of minutes of the launch of a space shuttle can be described very roughly as follows: The initial mass is 2×10^6 kg, the final mass (after 2 minutes) is about 1×10^6 kg, the average exhaust speed v_{ex} is about 3000 m/s, and the initial velocity is, of course, zero. If all this were taking place in outer space, with negligible gravity, **(a)** what would be the shuttle's speed at the end of this stage? **(b)** What is the thrust during the same period and how does it compare with the initial total weight of the shuttle (on earth)?

3. A massless spring is hanging vertically and unloaded, from the ceiling. A mass is attached to the bottom end and released. How close to its final resting position is the mass after 1 second, given that it finally comes to rest 0.5 meters below the point of release and that the motion is critically damped?

   Hint: use solution for critically damped oscillations in the form
   $$y(t) = (C_1 + C_2 t)e^{-\omega_0 t}$$

4. A mass m is suspended from a massless string, the other end of which is wrapped several times around a horizontal cylinder of radius R and moment of inertia I, which is free to rotate about a fixed horizontal axle. Using a suitable coordinate, set up the Lagrange equation of motion and find the acceleration of the mass m. [The kinetic energy of the rotating cylinder is $I\omega^2/2$.]

5. A bullet of mass m is fired with muzzle speed v_0 horizontally from a position at the North Pole. Find the direction and magnitude of the Coriolis force. How does the Coriolis force compare with the bullet's weight if v_0 = 1000 m/s?