PHYS 3220
FINAL EXAMINATION
Dec 15^{th}, 2006

Name: ______________________________

Student Number: ______________________________

Date: ______________________________

INSTRUCTIONS

- Write your name on this questionnaire.
- Do not un-staple this questionnaire.
- This final examination lasts 2 hours
- You are allowed to have your pen, and pencil/eraser only. No books or class notes allowed.
- **DO EACH QUESTION.**
- This examination questionnaire has 17 pages in total with 5 questions. Make sure your questionnaire has all 17 pages.
- Make sure that you spend the appropriate amount of time on each question
- The percent value of each question and the maximum amount of time that should be dedicated is indicated on the left of the question number.
- If you have any questions during the examination, raise your hand and I will come see you.
- You may not leave the examination room before the first 15 mins of the exam or 15 mins before the end of the exam. You may leave the exam at any other time.
- At the end of the exam stay in your seat and I will come pick up your exam.

Do not turn over this page until instructed to do so.

Good luck…

Questions	Mark
Question 1	
Question 2	
Question 3	

16%
(19.2 min) Question 1

a) Identify whether the following quantities are vectors or scalars. Circle the correct answer.

i)	Momentum	vector \ scalar
ii)	Speed	vector \ scalar
iii)	Displacement	vector \ scalar
iv)	Kinetic Energy	vector \ scalar
v)	Length	vector \ scalar
vi)	Angular Momentum	vector \ scalar
vii)	Torque	vector \ scalar
viii)	Potential Energy	vector \ scalar

b) If $\mathbf{a} = (a_x, a_y, a_z)$ and $\mathbf{b} = (b_x, b_y, b_z)$ are two vectors then compute the following:

i) $\vec{a} \cdot \vec{b}$

ii) $\vec{a} \times \vec{b}$

iii) $\nabla \vec{a}$

iv) $\nabla \times \vec{b}$

v) $\vec{a} + \vec{b}$

Question 1 (Continued)

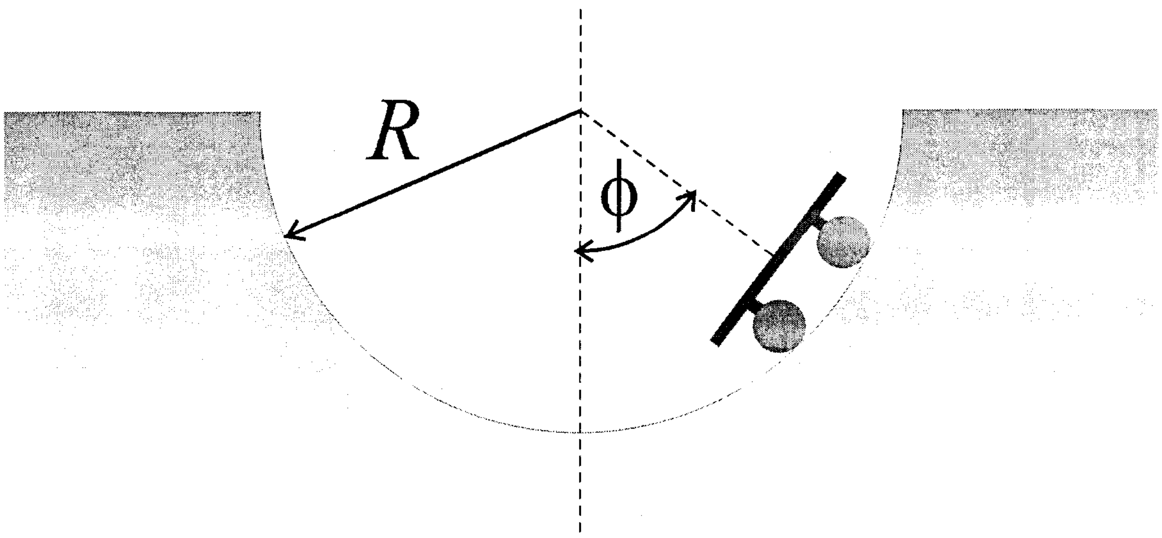
c) Derive an equation for the differential element dV in spherical coordinates.

d) Use the Taylor Series expansion $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ to show that any potential energy function $U(x)$ can be, to a first approximation, represented by a harmonic potential

32%
(38.4 min)

Question 2

Consider a skateboard in a half pipe of radius R as shown below.



***** Do not use Lagrangian Mechanics here in Question 1 *****

a) Derive an equation for the acceleration of the skateboard in polar coordinates.

b) If m is the mass of the skate board, show that the force on the skateboard is given by

$$F = m\left(-R\dot{\phi}^2, R\ddot{\phi}\right)$$

c) In the equation above, identify which term is the radial force and which term is the tangential force.

d) Derive an equation for the normal force that the half-pipe exerts on the skateboard. Draw a plot of the normal force as a function of ϕ for $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$

e) Show that the equation of motion of the skateboard in the half pipe is given by:

$$\ddot{\phi} = -\frac{g}{R}\sin(\phi)$$

f) Make a small angle approximation and show that $\phi(t) = A\sin(\omega t) + B\cos(\omega t)$ is a solutions. What is ω?

g) Assuming the skateboard starts at ϕ_o at $t = 0$ with an initial velocity $-v_o$ then find the equation of motion of the skateboard.

h) Make a plot that shows the motion of the skateboard as a function of time. Indicate on this plot the period of oscillation, maximum amplitude and other important information.

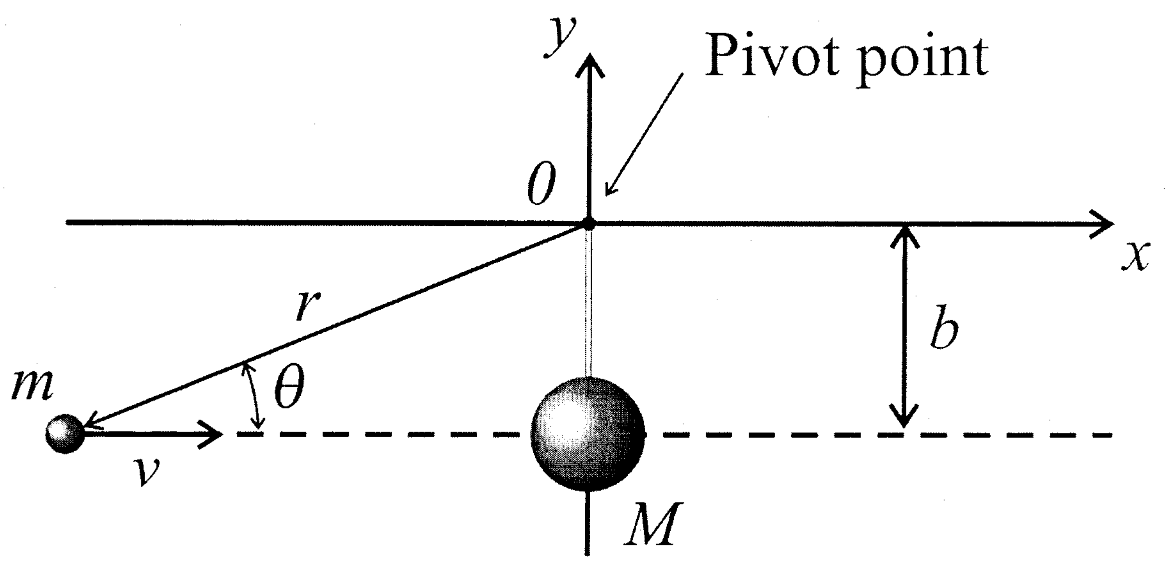
20%
(24 min) Question 3

Consider a sphere of mass M and radius R initially at rest suspended by a massless rod. The center of the sphere is at a distance b from the pivot point. The rod is connected by a frictionless pivot which allows the pendulum to rotate about that point. The pivot point is at the center at the origin of a Cartesian coordinate system (see figure below).

A lump of putty of mass m is thrown with a speed v towards the sphere at a distance b below the x-axis. When the putty hits the disk, it sticks to the disk and the two rotate with an angular frequency ω. Show that ω is of the form?

$$\omega = \frac{b\alpha v}{\beta R^2 + \gamma b^2}$$

Find α, β, and γ.

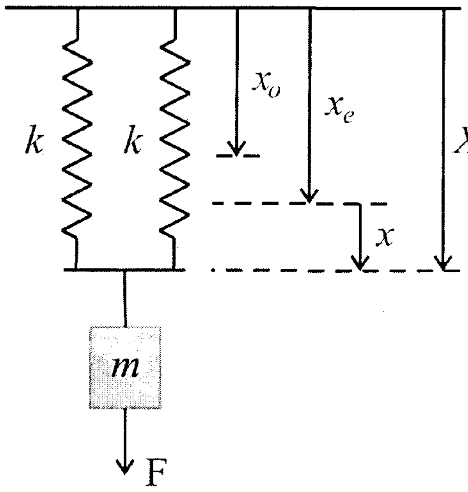


Use the parallel axis theorem $I = I_{cm} + mh^2$ to calculate the moment of inertia of the combined putty/pendulum system. Assume the putty is a point mass.

16%
(19.2 min)

Question 4

Consider a mass m suspended from two springs of identical spring constants k. The natural spring length of each spring is x_o. When the mass is hung on the springs, the springs elongate to a new equilibrium position x_e. A force **F** then pulls the mass down a distance x_i (not shown) from x_e. At time $t = 0$ the block is released.



a) Write down the differential equation of motion of this system in terms of k, x_o, X and m.

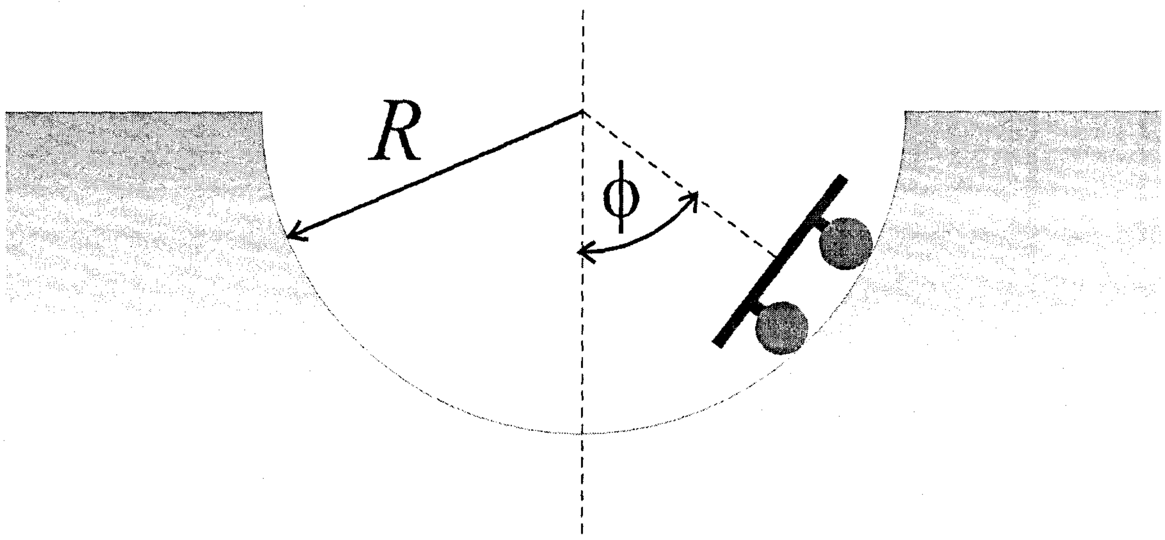
b) Make the proper change of variable so that your differential equation resembles that of a simple harmonic oscillator.

c) Solve the equation of motion based on the initial condition described above.

d) How would the angular frequency of the system be changed if the two springs would be replaced by one spring of spring constant k.

16%
(19.2 min) Question 5

We consider again a skateboard in a half pipe of radius R as shown below. However in this case we analyze this problem using the Lagrange formulism.



a) Write the Lagrangian for this system.

b) Show that the Lagrange equation $\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}$ leads to the differential equation:

$$\ddot{\phi} = -\frac{g}{R}\sin(\phi)$$

c) This problem is clearly a constrained system with $r = R$. Write the modified Lagrange equation:

$$\frac{\partial L}{\partial q} + \lambda\frac{\partial f}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}},$$

d) Which force of constraint λ is equal to?

Formula Sheet

$F = ma$ $\quad F = -kx$ $\quad w = mg$ $\quad F = \dfrac{GmM}{r^2}$ $\quad f_k = \mu N$ $\quad F = \dfrac{mv^2}{r}$ $\quad \vec{F} = -\nabla U$

$\vec{F} = \dfrac{d\vec{p}}{dt}$ $\quad \vec{I} = \Delta\vec{p}$ $\quad p = mv$

$$v = v_o + at$$
$$x = x_o + \tfrac{1}{2}(v_o + v)t$$
$$x = x_o + v_o t + \tfrac{1}{2}at^2$$
$$v^2 = v_o^2 + 2a(x - x_o)$$

$$\theta = s/r$$
$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T}$$
$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \omega_o + \alpha t$$
$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$v = \omega r$$
$$a_r = \omega^2 r$$
$$a_t = \alpha r$$

$$r_{CM} = \frac{\sum m_i r_i}{M}$$
$$r_{CM} = \frac{1}{M}\int r\,dm$$
$$v_{CM} = \frac{\sum m_i v_i}{M}$$

$$K = \tfrac{1}{2}I\omega^2$$
$$I = \sum m_i r_i^2$$
$$I = \int r^2 dm$$
$$I = I_{CM} + Mh^2$$

$$\tau = I\alpha$$
$$\vec{\tau} = \vec{r}\times\vec{F}$$
$$P = \tau\omega$$

$$\vec{l} = \vec{r}\times\vec{p}$$
$$l = I\omega$$
$$\tau = \frac{dl}{dt}$$

$U = mgh$ $\quad U = \tfrac{1}{2}k(\Delta x)^2$ $\quad T = \tfrac{1}{2}mv^2$ $\quad W = Fd$ $\quad P = Fv$

Moment of Inertia Formula