Name:______________________________
Student number:____________________

DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

Final Exam	**Physics 3000**	**December 11, 2012**
Fall 2012		**9:00-11:00**

INSTRUCTIONS:

1. Answer all seven (7) questions. The marks for each question are indicated. The total number of marks on the paper is 100. Budget time accordingly.

2. Write answers in the spaces provided. If additional space is needed, use the backs of sheets but indicate clearly where your answer is continued.

3. Where appropriate, units must be included for answers to be considered complete.

4. A list of useful formulae, a table of constants, and tables of semiconductor properties are attached at the end of the paper (last three pages). For your convenience, three particularly useful parameters are listed below.

Particularly useful parameters:

Relative permittivity (dielectric constant) for silicon: $\epsilon_{\mathrm{Si}} = 11.7$

Relative permittivity (dielectric constant) for silicon dioxide: $\epsilon_{\mathrm{SiO_2}} = 3.9$

Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-14}$ F/cm

Intrinsic carrier concentration for silicon at 300 K: $n_{i,\,\mathrm{Si}} = 1.5 \times 10^{10}$ cm^{-3}

Thermal voltage $\left(V_t = \frac{kT}{e}\right)$ at 300 K: $V_t(300\ \mathrm{K}) = 0.0259$ V

Electron charge: $e = 1.6 \times 10^{-19}$ C

Planck's constant: $h = 6.625 \times 10^{-34}$ J-s = 4.135×10^{-15} eV-s

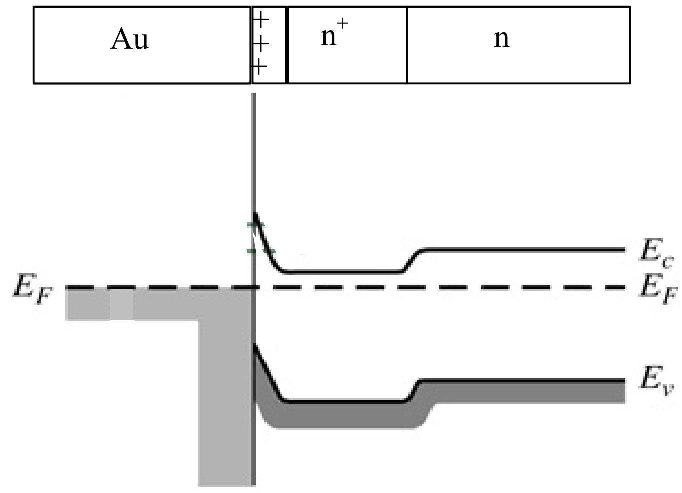
$c = 2.998 \times 10^{10}$ cm/s

Quadratic equation: roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1	2	3	4	5	6	7	Total

1. [11 marks]

The diagram to the right shows energy bands for a junction between gold and n-type silicon with a region of highly doped silicon at the interface. Assume that the work function for gold is $\phi_m = 5.1\,\text{V}$ and that the electron affinity for Si is $\chi = 4.01\,\text{V}$.



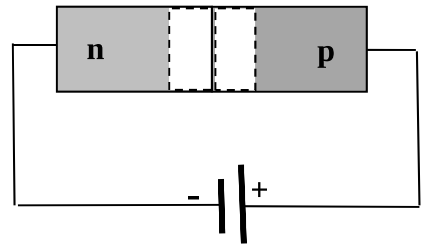
(a) **(4 marks)** Calculate the Schottky barrier energy, $e\phi_{B0}$, at the gold-Si interface **and** clearly indicate the height and location of the Schottky barrier **on the diagram**.

(b) **(4 marks)** What is the minimum doping concentration required in the n^+ region in order for the width of the space charge region to be less than 100 Å (i.e. 10^{-6} cm)? Assume that N_d is large enough in the n^+ region that you can assume $\phi_{B0} >> \phi_n$ where $e\phi_n = E_c - E_F$.

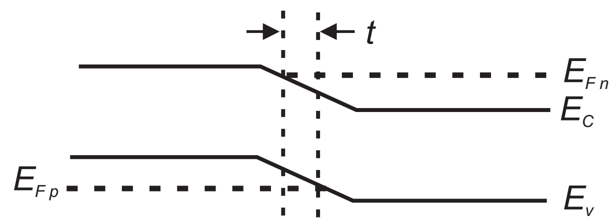
(c) **(3 marks)** Briefly explain why a junction like this might be useful for making an ohmic contact between a metal contact and a semiconductor.

2. [12 marks]

(a) **(5 marks)** The picture to the right shows a pn junction that is to be operated as a light-emitting-diode (LED). Briefly explain the process by which light is generated in the LED. Be sure to indicate which region(s) are involved and to indicate how majority or minority carriers are involved in the process occurring in those region(s).



(b) The energy-band diagram to the right represents a forward-biased pn junction.



(i) **(2 marks)** Briefly explain what aspect(s) of this diagram show(s) that the p-type and n-type materials in this device are degenerately doped.

(ii) **(3 marks)** Briefly explain why there is a population inversion (i.e. higher concentration of electrons in the conduction band than in the valence band) in the central region of the junction (labeled as thickness *t*).

(iii) **(2 marks)** What kind of device depends on the presence of a population inversion for its operation?

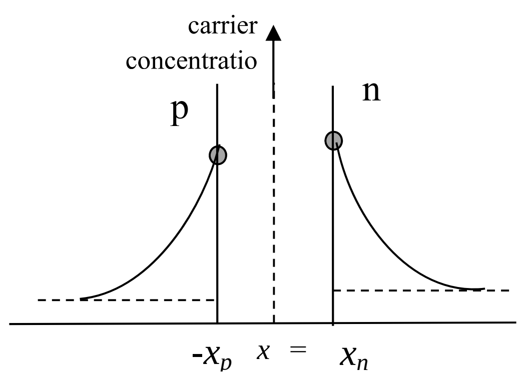
3. [14 marks]

(a) Assume that 1.49 eV photons are incident on the surface of a GaAs slab with an intensity $I_{\nu 0} = 0.09\ \mathrm{W/cm^2}$.

(i) **(3 marks)** If the absorption coefficient for 1.49 eV photons in GaAs is $\alpha = 4.4 \times 10^3\ \mathrm{cm^{-1}}$, what is the photon intensity at a depth of $x = 2.0 \times 10^{-4}$ cm below the GaAs surface?

(ii) **(3 marks)** What is the electron-hole pair generation rate at that depth? (Hint: be careful with units.)

(b) The dots on this diagram represent minority-carrier concentrations at the space charge edges for a forward-biased pn junction. The horizontal dashed lines represent thermal equilibrium minority-carrier concentrations.



(i) **(2 marks)** Attach the labels n_{p0}, p_{n0}, $n_p(-x_p)$, and $p_n(x_n)$ to the appropriate features on the diagram.

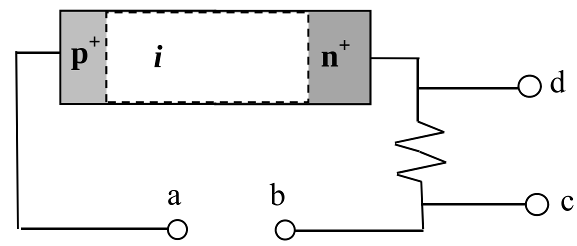
(ii) **(3 marks)** Assume that the semiconductor is Si and that the impurity concentrations on the p-type side and n-type side, respectively, are $N_a = 5.0 \times 10^{16}\ \mathrm{cm^{-3}}$ and $N_d = 2.0 \times 10^{16}\ \mathrm{cm^{-3}}$. Calculate the **thermal equilibrium minority-carrier** concentration on the **p-type** side assuming complete impurity ionization and $T = 300$ K.

(iii) **(3 marks)** Calculate the **minority-carrier** concentration at the **space charge edge** on the p-type side, $x = x_p$, if a forward-bias of 0.6 V is applied to this junction.

 Name:______________________________
Student number:____________________

4. [15 marks]

The drawing to the right represents a PIN junction diode operated as a photodiode.



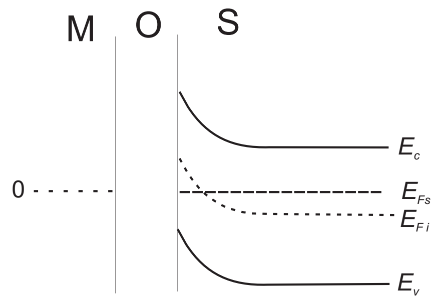
(a) **(4 marks)** Briefly explain why there is an induced electric field in the intrinsic region of this device and indicate the direction of this electric field on the diagram.

(b) **(4 marks)** For this device to operate as a photodiode, should terminal *a* be biased positive or negative with respect to terminal *b*? Briefly justify your answer and indicate, on the diagram, the direction of the current that will flow when the device is illuminated.

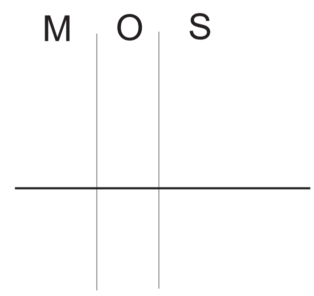
(c) **(4 marks)** Briefly explain the difference between prompt photocurrent and delayed photocurrent. In which regions of the device are the carriers contributing to the prompt and delayed photocurrent generated?

(d) **(3 marks)** Briefly comment on how the presence of an intrinsic region affects the properties of a PIN photodiode in comparison to those of a pn junction photodiode.

5. [15 marks]



a



b

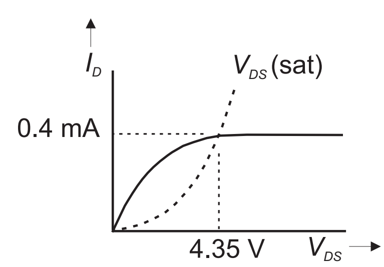
Diagram (a) above shows the energy band diagram on the semiconductor side of a metal-oxide-semiconductor (MOS) capacitor at **threshold bias**.

(a) **(2 marks)** Is the semiconductor p-type or n-type? Briefly justify your answer.

(b) **(2 marks)** Is the voltage applied to the metal gate positive or negative? Briefly justify your answer and indicate the Fermi level of the metal gate on diagram (a).

(c) **(3 marks)** On diagram (b), carefully draw the charge distributions on the metal gate and in the semiconductor in this bias state. You should show the charge density on the metal, Q'_m, the space charge density in the semiconductor, and any charge density induced in the semiconductor.

(d) The diagram to the right shows an *IV* curve, corresponding to $I_D(\text{sat}) = 0.4\,\text{mA}$ and $V_{DS}(\text{sat}) = 4.35\,\text{V}$, for a n-channel MOSFET with a threshold voltage of 0.65 V.



(i) **(2 marks)** What is the gate voltage corresponding to this curve?

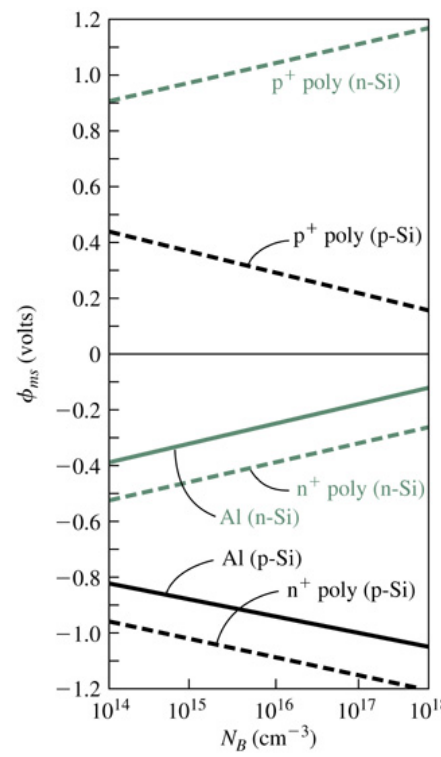
(ii) **(3 marks)** What is the conduction parameter, K_n, for this device?

(iii) **(3 marks)** What is the current I_D if $V_G = 4.0\,\text{V}$ and $V_{DS} = 3.0\,\text{V}$?

 Name:______________________________
Student number:____________________

6. [16 marks]

An enhancement mode, n-channel MOS transistor is fabricated on a **p-type silicon** substrate with $N_a = 10^{16}\text{cm}^{-3}$. The gate material is n^+ **polycrystalline silicon**. The threshold voltage is $V_T = +1.2\text{ V}$. Relative permittivities are given on the front page of the exam. Assume $T = 300\text{K}$.



(a) **(2 marks)** Using the graph to the right, estimate the metal-semiconductor work function potential difference, ϕ_{ms}, for this device.

(b) **(3 marks)** What is the potential difference $\phi_{Fp} = \frac{1}{e}(E_F - E_{Fi})$ between the Fermi energy and the intrinsic Fermi level in the substrate? Assume complete ionization?

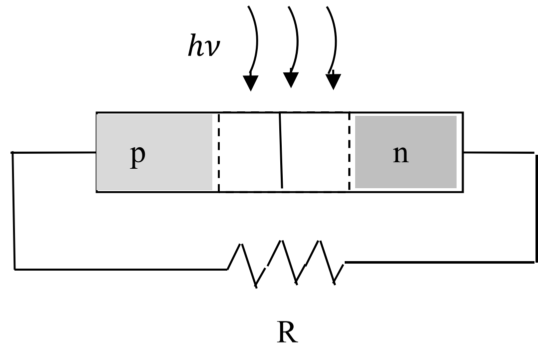
(c) **(3 marks)** What is the maximum space charge width, x_{dT}, in the semiconductor?

(d) **(2 marks)** What is the magnitude of the maximum space charge density per unit area, $|Q'_{SD}(\text{max})|$, in the depletion region of the semiconductor?

(e) **(3 marks)** If the equivalent fixed oxide charge is $Q'_{ss} = +8 \times 10^{-9}\text{ C/cm}^2$, what is the oxide capacitance per unit area?

(f) **(3 marks)** What is the oxide thickness?

 Name:______________________________
Student number:____________________



7. [17 marks] The diagram shows a pn junction solar cell connected to a resistive load. When illuminated, the **net** current in this device is $I = I_L - I_F$ where I_L is the photocurrent and I_F is the forward bias current.

(a) **(3 marks)** On the diagram, indicate the directions of I , I_L, I_F .

(b) **(3 marks)** Briefly explain how the photocurrent is generated and why it flows in the direction you have indicated.

(c) **(3 marks)** **Briefly** explain what causes the forward-bias current and why it flows in the direction you have shown.

(d) **(4 marks)** A silicon pn junction solar cell is illuminated such that the photocurrent is $I_L = 0.42$ A at $T = 300$ K . Its open circuit voltage is found to be 0.6 V under these conditions. What is its reverse saturation current. (Hint: recall that for a forward-biased pn junction, the current is $I = I_S \left[\exp\left(\frac{eV_D}{kT}\right) - 1\right]$.)

(e) **(4 marks)** The bandgap in silicon is 1.12 eV. Would you expect this solar cell to work better when illuminated by electromagnetic radiation with wavelength $\lambda = 0.8$ μm or when illuminated by electromagnetic radiation radiation with wavelength $\lambda = 1.4$ μm? Justify your answer. (Hint: you may find it useful to calculate the wavelength of a 1.12 eV photon.)

 Name:______________________________
Student number:____________________

Potentially useful formulae

Photons: $E_{\text{photon}} = h\nu$; $p = h/\lambda$; $c = \nu\lambda$; Bohr Atom: $E_n = -\frac{m_0 e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2}$; $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_0 e^2}$

Drift curr. density: $J_n = -e\sum_{i=1}^{n} v_i$; Eff. mass: $F_{\text{ext}} = m^* a$; Free particle: $E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$

Densities of states: $g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$; $g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$

F-D prob. function: $f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}$; M-B approx.: $f_F(E) \approx \exp\left[-\frac{(E-E_F)}{kT}\right]$

Carrier distributions: $n(E) = g_c(E) f_F(E)$; $p(E) = g_v(E)[1 - f_F(E)]$

Effective densities of states: $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$; $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$

Therm. equil. carrier concentrations.: $n_0 = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$; $p_0 = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$

Intrins. carr. concentration & Fermi level: $n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$; $E_{Fi} - E_{\text{midgap}} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$

Extrinsic semiconductor relationships: $n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$; $p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$

Fundamental semiconductor equation: $n_0 p_0 = n_i^2$

Donor electron relationships: $n_d = N_d - N_d^+ = \frac{N_d}{1+\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$; $\frac{n_d}{n_d + n_0} = \frac{1}{1+\frac{N_c}{2N_d}\exp\left[\frac{-(E_c - E_d)}{kT}\right]}$

Acceptor hole relationships: $p_a = N_a - N_a^- = \frac{N_a}{1+\frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)}$; $\frac{p_a}{p_a + p_0} = \frac{1}{1+\frac{N_v}{gN_a}\exp\left[\frac{-(E_a - E_v)}{kT}\right]}$

Compensated semiconductors, complete ionization:

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \qquad p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

Fermi energy level relationships:

$$E_c - E_F = kT\ln\left(\frac{N_c}{n_0}\right) \; ; \; E_F - E_v = kT\ln\left(\frac{N_v}{p_0}\right) \; ; \; E_F - E_{Fi} = kT\ln\left(\frac{n_0}{n_i}\right) = -kT\ln\left(\frac{p_0}{n_i}\right)$$

Conduct. & resist.: $e(\mu_n n + \mu_p p) = \sigma = \frac{1}{\rho}$ Mobility: $\mu_p = \frac{v_{dp}}{\varepsilon} = \frac{e\tau_{cp}}{m_p^*}$; $\mu_n = \frac{v_{dn}}{\varepsilon} = \frac{e\tau_{cn}}{m_n^*}$

Drift curr. dens.: $J_{drf} = e(\mu_n n + \mu_p p)\varepsilon_x$ Diffusion curr. dens.: $J_{dif} = eD_n\frac{dn}{dx} - eD_p\frac{dp}{dx}$

Total current density: $J = J_{drf} + J_{dif}$ Einstein relation: $\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$

Graded impurity distribution: $\varepsilon_x = -\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$

Equil. generation & recombination rates: $G_{n0} = G_{p0} = R_{n0} = R_{p0}$; $R_{n0} = \frac{n_0}{\tau_{n0}}$; $R_{p0} = \frac{p_0}{\tau_{p0}}$

Excess carrier recombination (p-type): $\delta n(t) = \delta n(0)e^{-t/\tau_{n0}}$; $R_n' = R_p' = \frac{\delta n(t)}{\tau_{n0}}$; $\tau_{n0} = \frac{1}{\alpha_r p_0}$

Excess carrier recombination (n-type): $\delta n(t) = \delta n(0)e^{-t/\tau_{p0}}$; $R_n' = R_p' = \frac{\delta n(t)}{\tau_{p0}}$; $\tau_{p0} = \frac{1}{\alpha_r n_0}$

Hall effect: $\vec{F} = q[\vec{\varepsilon} + \vec{v_d} \times \vec{B}]$; $p = \frac{I_x B_z}{edV_H}$; $n = -\frac{I_x B_z}{edV_H}$; $\mu_p = \frac{I_x L}{epV_x Wd}$; $\mu_n = \frac{I_x L}{enV_x Wd}$

Built in potential barrier (pn junction): $V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right)$

 Name:____________________________
Student number:____________________

Potentially useful formulae (continued)

Poisson's equation: $\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_s} = -\frac{d\varepsilon(x)}{dx}$

Sp. charge width: $x_n = \left[\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a}{N_d}\right)\frac{1}{N_a+N_d}\right]^{1/2}$; $x_p = \frac{N_d}{N_a}x_n$; $W = \left[\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$

Electric field: $\varepsilon_{\max} = -\frac{2(V_{bi}+V_R)}{W}$ Junction Capacitance: $C' = \left[\frac{e\epsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right]^{1/2}$

Schottky barrier: $\phi_{B0} = (\phi_m - \chi)$ $V_{bi} = \phi_{B0} - \phi_n$

pn diode: $J_D = J_S\left[\exp\left(\frac{eV_D}{kT}\right) - 1\right]$

Schottky barrier diode: $J = \left[A^*T^2\exp\left(\frac{-e\phi_{B0}}{kT}\right)\right]\left[\exp\left(\frac{eV_D}{kT}\right) - 1\right] = J_{ST}\left[\exp\left(\frac{eV_D}{kT}\right) - 1\right]$

MOS depletion width (p-type): $x_{dT} = \left(\frac{4\epsilon_s|\phi_{Fp}|}{eN_a}\right)^{1/2}$; $e\phi_{Fp} = E_F - E_{Fi} = -kT\ln\left(\frac{N_a}{n_i}\right)$

MOS depletion width (n-type): $x_{dT} = \left(\frac{4\epsilon_s\phi_{Fn}}{eN_d}\right)^{1/2}$; $e\phi_{Fn} = E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right)$

Metal-semiconductor work function difference (p-type): $\phi_{ms} = \left[\phi'_m - \left(\chi' + \frac{E_g}{2e} + |\phi_{Fp}|\right)\right]$

Metal-semiconductor work function difference (n-type): $\phi_{ms} = \left[\phi'_m - \left(\chi' + \frac{E_g}{2e} - \phi_{Fn}\right)\right]$

Flat-band voltage: $V_{FB} = \phi_{ms} - \frac{Q'_{ss}}{C_{\text{ox}}}$ Maximum space charge density: $|Q'_{SD}(\max)| = eN_a x_{dT}$

Threshold voltage: $V_{TN} = \frac{|Q'_{SD}(\max)|}{C_{\text{ox}}} - \frac{Q'_{ss}}{C_{\text{ox}}} + \phi_{ms} + 2|\phi_{Fp}|$

MOSFET (general): $I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$; $K_n = \frac{W}{L}\frac{\mu_n C_{\text{ox}}}{2}$; $C_{\text{ox}} = \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}}$

MOSFET (saturation): $V_{DS}(\text{sat}) = V_{GS} - V_T$; $I_D(\text{sat}) = K_n(V_{GS} - V_{TN})^2$

Electron inversion charge density: $n_s = N_a\exp\left(\frac{\Delta\phi_s}{V_t}\right)$

Channel length mod.: $\Delta L = \sqrt{\frac{2\epsilon_s}{eN_a}}\left[\sqrt{|\phi_{FP}| + V_{DS}(\text{sat}) + \Delta V_{DS}} - \sqrt{|\phi_{FP}| + V_{DS}(\text{sat})}\right]$; $\frac{I'_D}{I_D} = \frac{L}{L-\Delta L}$

Mobility variation: $\varepsilon_{\text{eff}} = \frac{1}{\epsilon_s}\left(|Q'_{SD}(\max)| + \frac{1}{2}Q'_n\right)$; $\mu_{\text{eff}} = \mu_0\left(\frac{\varepsilon_{\text{eff}}}{\varepsilon_0}\right)^{-1/3}$

Forward-biased pn junction.:

minority carrier conc. at space charge edge: $n_p = n_{p0}\exp\left(\frac{V_a}{V_t}\right)$; $p_n = p_{n0}\exp\left(\frac{V_a}{V_t}\right)$

reverse saturation current density: $J_S = \left(\frac{e\sqrt{D_p}p_{n0}}{\sqrt{\tau_{p0}}} + \frac{e\sqrt{D_n}n_{p0}}{\sqrt{\tau_{n0}}}\right)$

common-base current gain: $\alpha = \frac{i_C}{i_E}$ common-emitter current gain: $\beta = \frac{i_C}{i_B}$

photon flux intensity: $I_v(x) = I_{v0}e^{-\alpha x}$ electron-hole pair generation rate: $g' = \frac{\alpha I_v(x)}{hv}$

pn junct. solar cell: $I = I_L - I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$; $V_{\text{oc}} = V_t\ln\left(1 + \frac{I_L}{I_S}\right)$; $\left(1 + \frac{V_m}{V_t}\right)\exp\left(\frac{V_m}{V_t}\right) = 1 + \frac{I_L}{I_S}$

photoconductor: $I_L = eG_L\tau_p(\mu_n + \mu_p)A\varepsilon$ photoconductor gain: $\Gamma_{\text{ph}} = \frac{\tau_p\varepsilon}{L}(\mu_n + \mu_p)$

pn photodiode prompt photocurrent: $J_{L1} = eG_L W$

PIN photodiode prompt photocurrent: $J_L = e\Phi_0(1 - e^{-\alpha W})$

 Name:______________________________
Student number:____________________

Constants and Conversions

$$N_A = 6.02 \times 10^{23}$$

$$k = 1.38 \times 10^{-23}\ \text{J/K} = 8.62 \times 10^{-5}\ \text{eV/K}$$

$$e = 1.60 \times 10^{-19}\ \text{C}$$

$$m_0 = 9.11 \times 10^{-31}\ \text{kg}$$

$$\epsilon_0 = 8.85 \times 10^{-14}\ \text{F/cm}$$

$$h = 6.625 \times 10^{-34}\ \text{J-s} = 4.135\times 10^{-15}\ \text{eV-s}$$

$$c = 2.998 \times 10^{10}\ \text{cm/s}$$

$$1\ \text{eV} = 1.6\times 10^{-19}\ \text{J}$$

Silicon, gallium arsenide, and germanium properties at *T* = 300 K

Property	Si	GaAs	Ge
Atoms/cm^3	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^3)	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (^{o}C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ (V)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Electron mobility, μ_n (cm^2/V-s)	1350	8500	3900
Hole mobility, μ_p (cm^2/V-s)	480	400	1900
Electron effective mass (DoS) $\left(\frac{m_n^*}{m_0}\right)$	1.08	0.067	0.55
Hole effective mass (DoS) $\left(\frac{m_p^*}{m_0}\right)$	0.56	0.48	0.37

Table 3.3 | Impurity ionization energies in silicon and germanium

Impurity	Ionization Energy (eV) Si	Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

Table 3.4 | Impurity ionization energies in gallium arsenide

Impurity	Ionization Energy (eV)
Donors	
Selenium	0.0059
Tellurium	0.0058
Silicon	0.0058
Germanium	0.0061
Acceptors	
Beryllium	0.028
Zinc	0.0307
Cadmium	0.0347
Silicon	0.0345
Germanium	0.0404