P2820 FINAL APRIL 11, 2007 9-11am

This is an open book final. Please rename this file Lastname_Firstname_FINAL.nb and submit your completed exam as you normally submit the labs. There are 4 question, for a total of 40 marks. Time allowed is 2 hours.

Q1 (12 marks)

Wiley Coyote attaches a miniature rocket to a massless horizontal Hooke's Law spring ($F_S = -k\,x$) with spring constant $k = $ 2N/m. The system is initially at rest and the spring is not displaced. At time $t = 0$, the rocket is ignited and produces a thrust in the x-direction of $T(t) = \text{Exp}(-t/2)$ N. As the rocket burns, its mass decreases and is given in kilograms by $M(t) = 0.3\,\text{Exp}(-t/2) + 0.1$. There is a drag force of $F = -0.2\,v$ N, where v is the velocity of the rocket. Plot the displacement of the rocket for 10 seconds.

Q2 (12 marks)

Download from the Final section of the course webpage the file TE.dat. It contains two columns: column 1 is temperature T in K, column 2 is energy E in J/mol for a particular liquid. Theory tells us that E(T) should have an inflection point, and so our fitting function must reflect this.

a) Import the data and plot them nicely

b) Fit the data with a cubic polynomial. Plot the fit on top of the data.

c) Plot the heat capacity from 2800K to 7000K. The heat capacity is $C = \frac{dE}{dT}$, which you get be differentiating your fit to the energy with respect to T.

Q3 (12 marks)

At time t=0 a metal bar 1 m long has a uniform temperature of T=10 °C. The left end (x=0) is insulated, while the right end is heated in such a way that the temperature at the right end is given by

```
Remove["Global`*"]
r[t_] = If[t < 10, 10 + t², 110]
Plot[r[t], {t, 0, 15}, AxesLabel → {"time [s]", "T at x=1 [K}"}];
```

The thermal diffusivity of the bar is not uniform. It is given in m^2/s by

```
a[x_] = 10⁻² (20 Exp[-20 (x - 0.5)²] + 1)
Plot[a[x], {x, 0, 1}, PlotRange → {{0, 1}, {0, 21 10⁻²}},
  AxesLabel → {"Position [m]", "Thermal diffusivity [m²/s]"}];
```

Solve the heat equation $\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}$ with appropiate boundary conditions to find and plot the temperature profile of the bar at t=50 s.

Q4 (4 marks)

A student stands in the centre of a large rotating ice sheet and slides a puck away from himself in the y direction with speed 1 m/s. This y direction is in the frame of reference of the student, i.e. it is a rotating frame of reference. From the student's point of view, the puck does not travel in a straight line, but rather appears to be subjected to an effective force,

$$\vec{F_r} = -m\,\vec{\omega} \times \left(\vec{\omega} \times \vec{r}\right) - 2\,m\,\vec{\omega} \times \vec{v}$$

where $\vec{\omega}$ is the angular velocity of the ice sheet, $\vec{r}$ is the position of the puck in the student's coordinate system and $\vec{v}$ is the puck's velocity in the student's coordinate system. The first term gives the "centrifugal" force, and the second term is the "coriolis" force. In our case the ice sheet is rotating counterclockwise at 0.1 rad/s when viewed from above , so

$$\vec{\omega} = \{0,\ 0,\ 0.1\}\ \text{rad} / \text{second}$$

Plot the trajectory of the puck in the (student's) xy plane for 60 seconds.