P2820 - Computational Mechanics - Fall 2007

FINAL EXAM - December 11, 9-11am, 40 marks

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Question 1 [20 marks]

For this question, you will need to download the file "spring.dat" from the website. It contains two columns of data, the first being time t in seconds, while the second column gives the displacement x(t) from equilibrium in metres of a mass attached to a spring sliding on a low-friction surface.

a) [5 marks] Import the data and plot the displacement as a function of time for the oscillator.

b) [7 marks] Write a function that calculates the acceleration of the oscillator directly from the displacement data. Plot the acceleration as a function of time.

c) [5 marks] The mass of the oscillator is 1.2 kg. Assuming that the force exerted by the spring is F=-kx (where x is displacement), plot two appropriate quantities in such a way that the slope of the graph should be k.

d) [3 marks] Because of the data are not perfect, there should be some scatter in your plot. Use linear regression to determine k along with an error estimate. Do not forget to include units.

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Needs["Statistics`LinearRegression`"]
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Question 2 [20 marks]

This question involves modelling a skyjumper falling through the air. Initially, air resistance is very small, but upon deployment of a parachute, air drag suddenly increases to a large value. The mass of the jumper is 60kg, acceleration due to gravity is 9.81 m/s^2, the jumper starts from a height of 1000m and opens the parachute at about 200m above the ground.

a) [3 marks] Parameters for our system as well as the function describing the drag coefficient (in kg/m) are given below. Plot the drag coefficient as a function of height (you may need to use PlotRange to get a good plot).

```
(* Parameters *)

m = 60; (* kg *)
g = 9.81; (* m/s^2 *)
h = 1000; (* initial height in m *)
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T = 80; (* estimate in s of how long it takes to reach the ground *)
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(* The drag coefficient as a function of height in metres *)
d = 10; (* metres *)
y0 = 200;  (* approximate height at which parachute deploys in metres *)
kd = 0.48;  (* initial drag coefficient in kg/m *)
kp =  30.24; (* parachute drag coefficient in kg/m *)
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k[y_] = kp - (kp - kd)/2 (1 + Tanh[(y - y0)/d]);
```

b) [5 marks] Assuming the air drag force is $F_d = -k\ |v|\ v$, use NDSolve to find the height y and velocity v as functions of time. Plot your results for y(t) and v(t).

c) [6 marks] Now solve for the height and velocity as functions of time using your own solver. Highest marks given in order of 4th order Runge-Kutta, Midpoint, and Euler. Use a timestep of 0.25s. Plot your results for y(t) and v(t).

d) [3 marks] Compare your results for y(t) and v(t) from parts b) and c) by plotting corresponding quantities on the same graph.

e) [3 marks] Determine the time it takes for the jumper to reach the ground using your results from part b). Repeat using results from part c). By what percentage do the results differ?