MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF PHYSICS AND PHYSICAL OCEANOGRAPHY

PHYSICS 2750 FINAL EXAM - FALL 2007 - December 13, 2007

NAME:____________________________ STUDENT NUMBER:____________________________

INSTRUCTIONS:

1. Put your name and student number on each page.
2. Do ALL 5 questions.
3. Some possibly useful equations and constants are provided on the next page.
4. Use only the paper provided. No other books, notes or papers are permitted.
5. Do not remove examination papers from the examination room.

CONSTANTS AND FORMULAE

$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + U(x)\psi(x) = E\psi(x)$

$< f(x) >= \int f(x)dP(x) = \int f(x)P(x)dx$

$e^{i\phi} = cos(\phi) + isin(\phi)$

$p = h/\lambda, \;\; E = hf$

$m_p = 1.673 \times 10^{-27} kg$

$m_n = 1.675 \times 10^{-27} kg$

$c = 3.00 \times 10^8 \; m/s$

$hc = 1240 eV \cdot nm$

$\hbar = 1.055 \times 10^{-34} \; J \cdot s$

$E^2 = p^2c^2 + (mc^2)^2$

$n = 1, 2, 3, ...; \;\; l = 0, 1, 2, ...(n-1) \; ; \;\; m_l = 0, \pm 1, \pm 2, ... \pm l$

$1 \; eV = 1.602 \times 10^{-19} \; J$

$E_n = -13.6(\frac{Z^2}{n^2}) \; eV$

$E = p^2/2m$

$m_{^{220}_{86}Rn} = 220.011368 \; u$

$m_{^{216}_{84}P0} = 216.001888 \; u$

$m_{^4_2He} = 4.002603 \; u$

$m_{^1_1H} = 1.007825 \; u$

$1 \; u = 1.66 \times 10^{-27} \; kg = 931.494 \; MeV/c^2$

$T_{1/2} = 0.693/\lambda$

$N(t) = N_0 e^{-\lambda t}$

$1Ci = 3.7 \times 10^{10}$ decays/s

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1. Wave-Particle Duality. (20 points.)

   (a) Explain the notion of a wave packet. How is it constructed mathematically ? How is it useful for understanding quantum particles ? Use sketches.

   (b) What is the difference between the *group velocity* and *phase velocity* ?

   (c) Double Slit Experiment. If the intensity of quantum particles is given by $I = \Psi\Psi^*$ where $\Psi = |\Psi|e^{i\phi}$ and $\Psi^* = |\Psi|e^{-i\phi}$ is the particle wavefunction, show that in the case of both slits open:

   $$I_{1+2} = I_1 + I_2 + 2|\Psi_1||\Psi_2|\cos(\phi_1 - \phi_2).$$

   What is the meaning of the last term?

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2. Quantum Mechanics in one dimension. (20 points.)

A particle of mass m moves in the region $-L < x < L$ under the influence of a potential energy given by

$$U(x) = -\frac{\hbar^2 x^2}{mL^2(L^2 - x^2)}.$$

The potential is infinite outside this region. The wavefunction which satisfies the Schrodinger equation is given by

$$\psi(x) = A(1 - \frac{x^2}{L^2}),$$

where $A = \sqrt{15/16}$ ensures normalization.

(a) Use this wavefunction in the Schrodinger equation and find the energy of this state in terms of $\hbar, m$ and L.

(b) What is the probability of finding the particle in the region $0 < x < L$?

(c) Consider the following general question (not restricted to the above problem). Explain how to answer "How big is it ?" for quantum objects. A sketch may be useful.

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3. Atomic Structure. (20 points.)

   (a) Explain Rutherford's experiment using α-particles. Use a sketch. How did it lead to the conclusion that atoms have a small massive nucleus?

   (b) Show how Bohr's quantization ($n\hbar$) of angular momentum $L = mvr$ and the balancing of the electron's electrostatic force $F_{es} = ke^2/r^2$ with its Centripetal force $F_C = mv^2/r$ leads to quantization of the electron's orbital radius $r = n^2\hbar^2/(mke^2)$.

   (c) What is the wavelength of the absorbed photon that can cause the electronic state of Li^{3+} to change from $n = 2$ to $n = 4$?

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4. Quantum Mechanics in three dimensions: Hydrogen-like atoms. (20 points.)

   (a) Explain the concept of energy *degeneracy* in quantum mechanics. Use results for the Hydrogen atom as an example.

   (b) The photon has orbital angular momentum $l = 1$. Atomic *Optical* transitions conserve orbital angular momentum. Sketch an energy (n,l) diagram for the three lowest energy states of Hydrogen and indicate the allowed optical transitions.

   (c) Oxygen has 8 electrons. Using Hund's rule of maximizing the total spin quantum number S, show the spin state (using up-arrows $\uparrow$ and down-arrows $\downarrow$) of the 8 electrons that go into the 1s, 2s and 2p orbitals.

   (d) Show that the expectation value of the electrostatic potential of the Hydrogen-like ions with Z protons and one electron $U = -Zke^2/r$ is given by $< U >= -ke^2Z^2/a_0 = 2\text{E}$ in the ground state where the wavefunction is given by

$$\psi = \frac{1}{\sqrt{\pi}} \frac{Z^{3/2}}{a_0^{3/2}} e^{-Zr/a_0}.$$

   Recall that for spherically symmetric wavefunctions, $dP = 4\pi r^2 |\psi|^2 dr$.
   Also $\int_0^\infty xe^{-x} dx = 1$.

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5. Nuclear Structure. (20 points.)

   (a) Explain in qualitative terms why $N > Z$ helps stabilize large Z atoms.

   (b) List three properties of neutrinos.

   (c) A sample of isotope ^{131}I, which has a half-life of 8.04 days, has an activity R (recall $R = |dN/dt|$) of 5.0 mCi at the time of shipment. Upon receipt in a medical laboratory, the activity has been reduced to 4.2 mCi. How much time has elapsed between the two measurements ?

   (d) The alpha decay scheme of $^{220}_{86}Rn$ is given by

$$^{220}_{86}Rn \;\rightarrow\; ^{216}_{84}Po + \,^{4}_{2}He$$

   Find the kinetic energy of the α particle assuming the daughter nucleus, $^{216}_{84}Po$, has zero recoil velocity.