Final exam. P2055 Winter 2007

1. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in Figure 1. You may assume that $l >> d$. Find an expression for the capacitance of the device in terms of the plate area A and d, κ_1, κ_2 and κ_3.

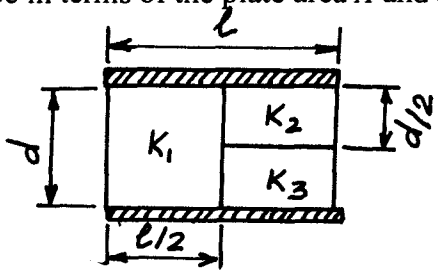

Figure 1.

2. The switch in Figure 2 is connected to point a for a long time. After the switch is thrown to point b, what are
   (a) the frequency of oscillation of the LC circuit,
   (b) the maximum charge that appears on the capacitor,
   (c) the maximum current in the inductor
   (d) the total energy the circuit possesses at t = 3 s?

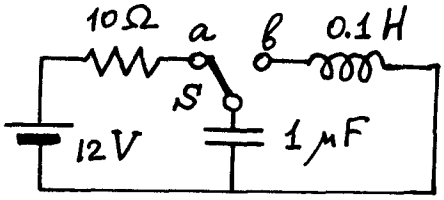

Figure 2.

3. Consider the filter circuit shown in Figure 3.
   (a) Find the ratio of the output voltage to the input voltage
   (b) What value does this ratio approach as the frequency decreases toward zero? What value does this ratio approach as the frequency increases without limit?
   (c) At what frequency is the ratio equal to one half?

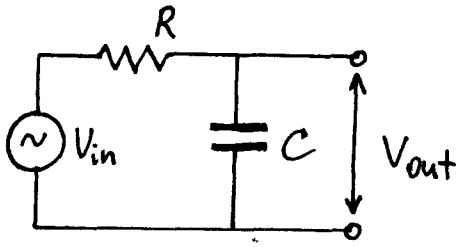

Figure 3.

4. Answer the following questions:
   (a) If the frequency is doubled in a series RLC circuit, what happens to the resistance, the inductive reactance, and the capacitive reactance?
   (b) If the resistance of the wires in an LC circuit were not zero, would the oscillations persist? Explain.
   (c) Embodied in Kirchhoff's rules are two conservation laws. What are they?
   (d) Two lightbulbs operate from 120 V. One has a power of 25 W and the other 100 W. Which bulb has higher resistance? Which bulb carries more current?

5. An electric current is given by the expression $I(t) = 100 \sin(120\pi t)$, where I is in amperes and t is in seconds. What is the total charge carried by the current from $t = 0$ to $t = (1/240)$ s?

Formulae

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan\varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$U_L = \frac{1}{2} L I^2$$

$$U_C = \frac{1}{2} \frac{Q^2}{C}$$

$$C = \kappa \frac{\varepsilon_0 A}{d}$$

$$R = \rho \frac{\ell}{A}$$

$$L = \frac{N \Phi_B}{I}$$

$$C_{eq} = C_1 + C_2 + \ldots$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

$$p = 2aq$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$I = nq v_d A$$

$$\vec{J} = \sigma \vec{E}$$

$$\rho = \rho_0 \left(1 + \alpha (T - T_0)\right)$$

$$R_{eq} = R_1 + R_2 + \ldots$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

$$L = \mu_0 \frac{N^2 A}{\ell}$$

$$M_{12} = \frac{N_2 \Phi_{12}}{I_1}$$