THE FORTY SECOND W.J. BLUNDON MATHEMATICS CONTEST

Sponsored by
The Canadian Mathematical Society *
in cooperation with
The Department of Mathematics and Statistics
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1. A bath can be filled with water running from two separate pipes controlled by two separate taps. One pipe supplies cold water and the other supplies hot water. If the drain plug is used, it takes 5 minutes to fill the bath with cold water when that tap is completely open, while it takes 10 minutes to fill the bath with hot water when its tap is completely open. If the bath is filled with water, it takes 6 minutes to drain the bath after taking out the plug. How much time does it take to fill the bath if both taps are completely open and the drain plug is not used? Express your answer in seconds.

   Solution: Let the bath contain 1 unit of water when completely filled. The speed of cold water supply is $\frac{1}{5}$ (units/minute). The speed of hot water supply is $\frac{1}{10}$ (units/minute). The speed of draining is $\frac{1}{6}$ (units/minute).

   The speed of water supply if both taps are open and the drain plug is not used is $\frac{1}{5}+\frac{1}{10}-\frac{1}{6}=\frac{2}{15}$ (units/minute). The time required to completely fill in the bath is $\frac{15}{2}=7.5$ minutes $=$ 420+30=450 seconds.

2. In the grid shown, 15 cells contain X's and 10 cells are empty. Any X may be moved to any empty cell. What is the smallest number of X's that must be moved so that each row and each column contains exactly three X's? Show that your answer is correct by showing the moves of the X's.

X	X	X	X	
X	X	X		X
X	X	X		
X	X			
		X	X	

   Solution: Since 3 columns contain 4 X's each, we need at least 3 moves.

   Use notation (n,m) to indicate the position in the n-th row and the m-th column. For example, (1,1) is the position in the top left corner and (1,5) is the position in the top right corner.

   It is possible to complete the task in 3 moves, for example, move X from (1,1) to (5,5), then from (2,2) to (4,5) and then from (3,3) to (3,4). Thus, the answer is 3.

	X	X	X	
X	X	X		X
X	X	X		
X	X			
		X	X	X

	X	X	X	
X		X		X
X	X	X		
X	X			X
		X	X	X

	X	X	X	
X		X		X
X	X		X	
X	X			X
		X	X	X

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3. (a) Find the largest *6-digit* number in which each digit starting from the 3rd is equal to the sum of two previous digits.

   (b) Find the largest whole number in which each digit starting from the 3rd is equal to the sum of two previous digits.

   Solution:

   (a) The digit are $x, y, x+y, x+2y, 2x+3y, 3x+5y$. We have $3x+5y < 10$. To maximize the number we take $x = 3$ and $y = 0$. So the answer is 303369.

   (b) To maximize the number we need more digits so we take the first digits as small as possible. The answer is 10112358.

4. A square was cut into six parts, each of which was a convex polygon. Five of these parts were lost and the one that remained had a shape of a regular octagon with side length 1 cm.

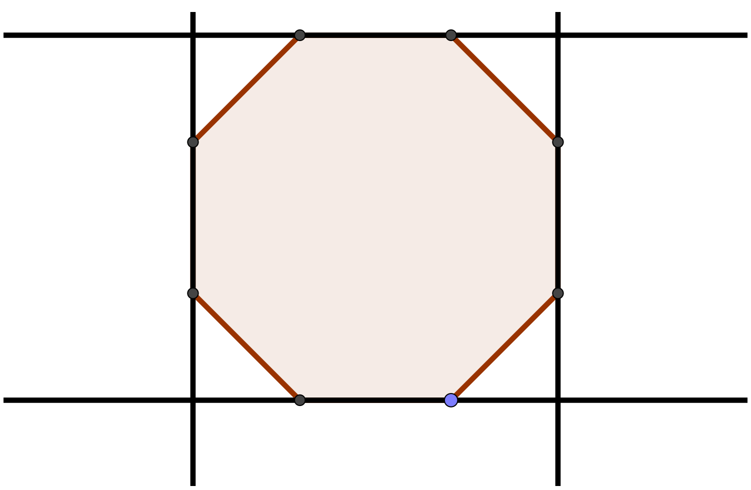
   (a) Is it possible to determine the exact side length (in cm) of the original square from the dimensions of the octagon? Is the answer unique?

   (b) Is it possible to determine the shape of the missing parts? Is the answer unique? Explain why.

   Solution:

   (a) Since each part was convex, it could have only one common side with the octagon. Four isosceles right triangles with hypothenuse 1 cm must be added to the octagon to form a square (see Figure 1). To account for six convex parts in total, one of the isosceles right triangles could be seen as composed from two convex parts.

   By the Pythagorean theorem, each leg of the isosceles right triangle with hypothenuse 1 cm is $\sqrt{1/2}$ cm. To have a bigger square, one would need at least two more parts (e.g. two rectangles), which is not consistent with the condition of the problem. Thus, the square has the smallest possible dimensions. The side length of the square is $1 + 2\sqrt{1/2} = (1+\sqrt{2})$ cm.



   (b) It is possible to describe the shapes of missing parts but not uniquely. For example, it could be three isosceles right triangles with hypothenuse 1 cm and the forth such triangle decomposed in two triangles or into a triangle and a quadrilateral.

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5. Find whole numbers a, b, c, d with the smallest possible values for b and d such that

$$\frac{80+\sqrt{120}+\sqrt{360}+\sqrt{19200}}{\sqrt{10}+\sqrt{30}} = a\sqrt{b}+c\sqrt{d}.$$

Solution:

$$\begin{aligned}
&\frac{80+\sqrt{120}+\sqrt{360}+\sqrt{19200}}{\sqrt{10}+\sqrt{30}} \\
&= \frac{\sqrt{6400}+\sqrt{12\times 10}+\sqrt{12\times 30}+\sqrt{19200}}{\sqrt{10}+\sqrt{30}} \\
&= \frac{\sqrt{640\times 10}+\sqrt{12\times 10}+\sqrt{12\times 30}+\sqrt{640\times 30}}{\sqrt{10}+\sqrt{30}} \\
&= \frac{(\sqrt{12}+\sqrt{640})(\sqrt{10}+\sqrt{30})}{\sqrt{10}+\sqrt{30}} \\
&= \sqrt{12}+\sqrt{640} \\
&= 2\sqrt{3}+8\sqrt{10}
\end{aligned}$$

6. There are 500 lamps numbered from 1 to 500, each of which could be turned either ON or OFF. We say that the *state of a lamp is changed* if it is changed either from ON to OFF or from OFF to ON. Originally all lamps are turned OFF. The first person changes the state of all lamps (that is, turned all of them ON). Then the second person changes the state of each lamp with even number. Then the third person changes the state of each lamp whose number is a multiple of 3, etc.

   All in all, the k-th person changes the state of each lamp whose number is a multiple of k for all $1 \le k \le 500$. How many lamps are emitting light as the result of this procedure?

   Solution:

   All numbers except perfect squares have even number of divisors (for example, 6 has four divisors: 1, 2, 3, and 6), so a lamp with such number will be turned OFF in the end of this process.

   Each perfect square has odd number of divisors (for example, 4 has 3 divisors: 1, 2 and 4), so the lamps with these numbers will be ON.

   Note that $22^2 = 484 < 500$ but $23^2 = 523 > 500$. Thus, the number of perfect squares less than 500 is 22, so the answer 22.

7. Let $ABCD$ be a square with side length a. Arcs of circles with radius a and centres at each vertex A, B, and D are drawn. The arcs intersect at points E and F. Find the ratio of areas of the square $ABCD$ and triangle AEF.

   Solution: If A is placed at $(0, 0)$, then D is at $(a, 0)$. Two circles centred at A and D respectively have equations $x^2 + y^2 = a^2$ and $(x-a)^2 + y^2 = a^2$. Subtracting first equation from the second, we obtain $x = a/2$ and so $y = a\sqrt{3}/2$. These are coordinates of the intersection point E. By symmetry, F has coordinates $(a\sqrt{3}/2, a/2)$.

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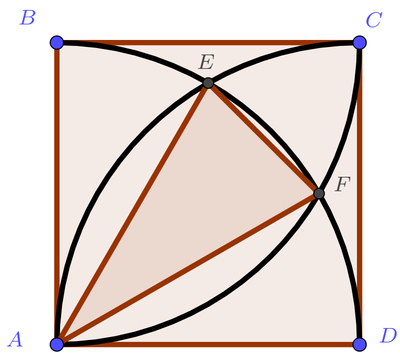
The area of AEF is $\frac{1}{2}|F_xE_y - F_yE_x| = \frac{1}{2}(\frac{3a^2}{4} - \frac{a^2}{4}) = \frac{a^2}{4}$.

One can alternatively argue that $\angle DAF = \angle BAE = \angle FAE = 30°$. This is done as follows. Call M the foot of perpendicular dropped from F to AD. Then AMF is a right triangle with $MF = \frac{1}{2}CD = \frac{1}{2}AD$, so $\angle DAF = \angle MAF = \arcsin\frac{1}{2} = 30°$. By the same argument, $\angle BAE = 30°$. Then $\angle FAE = 90° - \angle BAE - \angle DAF = 30°$.

Thus, the area of AEF is $\frac{1}{2}a^2\sin(30°) = \frac{a^2}{4}$.

Thus, the ratio of the areas is 4.



8. In some countries a postal code consists of 2 letters followed by 4 digits. In one of these countries there are 30 letters and 10 digits to choose from.

   (a) What is the probability that the postal code contains repeated letters and no repeated digits (e.g. AA1234)? Express your answers as a fraction in lowest terms.

   (b) Is it more likely or more unlikely that at least one symbol (letter or digit) in a postal code is repeated? Justify your answer.

   Solution:

   (a) The probability is

$$\frac{30 \cdot 1 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{30 \cdot 30 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{3 \cdot 7}{10 \cdot 5 \cdot 5 \cdot 5} = \frac{21}{1250}$$

   (b) The probability that there are no repetitions is $\frac{30 \cdot 29 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{30 \cdot 30 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$.

   The probability that at least one symbol is repeated is

$$1 - \frac{30 \cdot 29 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{30 \cdot 30 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = 1 - \frac{29 \cdot 21}{1250} = \frac{641}{1250} \geq \frac{1}{2}.$$

   So, it is more likely that at least one symbol is repeated.

9. Four numbers p, q, r, s satisfy $p < q < r < s$. Each of the six possible pairs of distinct numbers has a different sum. The four smallest sums are $a < b < c < d$. What is the sum of all possible values of s in terms of a, b, c, d?

   Solution:

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The smallest sums are $p+q=a$ and $p+r=b$. The largest sums are $s+r$ and $s+q$. The middle sums are $q+r$ and $p+s$. We need to consider two cases:

(1) $q+r=c$ and $p+s=d$;

(2) $q+r=d$ and $p+s=c$.

In case (1), from the two smallest sums we have $r-q=b-a$, so $r=q+b-a$. Substitute in $q+r=c$ to get $q=(c-b+a)/2$. Substitute in $p+q=a$ to get $p=(a+b-c)/2$. So, from $p+s=d$, we obtain $s=d-p=(2d+c-b-a)/2$.

Case (2) is obtained from case (1) by swapping c and d. Here $s=(2c+d-b-a)/2$.

Thus, the sum of possible values of s is $\frac{3d+3c-2b-2a}{2}=\frac{3}{2}(d+c)-(b+a)$.

10. Find all integer solutions, if any exist, to equation $15x^2-7y^2=9$.

Solution:

Note that $15x^2-5y^2$ is always divisible by 5, so we can ask if there are any values of y such that $-2y^2$ has the same remainder when divided by 5 as 9 does, that is 4.

In other words, we consider the equation modulo 5: $-2y^2 \equiv 4 \pmod 5$.

Then, multiply both sides by 2 to get $-4y^2 \equiv 8 \pmod 5$.

Since $-4 \equiv 1 \pmod 5$ and $8 \equiv 3 \pmod 5$, we must have $y^2 \equiv 3 \pmod 5$.

Calculate quadratic residues mod 5:

$y \equiv 0 \pmod 5$	$y^2 \equiv 0 \pmod 5$
$y \equiv 1 \pmod 5$	$y^2 \equiv 1 \pmod 5$
$y \equiv 2 \pmod 5$	$y^2 \equiv 4 \pmod 5$
$y \equiv 3 \pmod 5$	$y^2 \equiv 9 \pmod 5 \equiv 4 \pmod 5$
$y \equiv 4 \pmod 5$	$y^2 \equiv 16 \pmod 5 \equiv 1 \pmod 5$

The only possible values for $y^2 \pmod 5$ are 0, 1, and 4. The value 3 is not a quadratic residue modulo 5. Thus, the original Diophantine equation has no solutions.

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