Mathematics 4370
Number Theory

Does the quadratic equation $x^2 - 1141y^2 = 1$ have a solution with x and y positive integers? Otherwise put, is $1141y^2 + 1$ ever a perfect square? This may be checked experimentally. It turns out that the answer is no for all positive y less than one million. Experimenting further, we discover that the answer is still no for all y less than one trillion (one million million, or 10^{12}). We go overboard and check all y up to one trillion trillion (10^{24}). Again the answer is, no. Most would probably conclude that there is no positive y such that $1141y^2 + 1$ is a perfect square. But there is! In fact there are infinitely many such y, the smallest among them having 26 digits. This is a fairly trivial result after one has studied continued fractions.

Another fascinating problem is Fermat's Last Theorem (stated 300 years ago and proof announced, with much fanfare, in the summer of 1993), which asserts that the equation $x^n + y^n = z^n$ has no nontrivial solutions in integers if n is greater than two. To address this problem, we first study *quadratic domains*, rings that share many properties of the integers but in which not all elements factor uniquely as products of primes.

Text. There has been no formal text, but one of the most widely used references is *An Introduction to the Theory of Numbers* by I. Niven and H. S. Zuckerman.

Marks. A typical scheme is to assign 50% of the final grade in the course to a final examination, 40% to a midterm and 10% to homework.

Calendar description. **4370 Number Theory** examines continued fractions, an introduction to Diophantine approximations, selected Diophantine equations, the Dirichlet product of arithmetic functions, the quadratic reciprocity law, and factorization in quadratic domains.
Prerequisite: Mathematics 3370.

Offered. Occasionally