MATHEMATICS 4341
COMBINATORIAL DESIGNS

In 1782, the Swiss mathematician, Léonhard Euler, proposed the following problem:

> Thirty-six officers of six ranks and from six different regiments are to march in a square formation of size 6 × 6. Each row and each column of the formation is to contain exactly one officer of each rank and exactly one from each regiment. Is such a formation possible?

The existence of such a formation is equivalent to the existence of a pair of *orthogonal* 6 × 6 *Latin squares*. Euler could easily construct such pairs of squares of order three, four and five, but he had trouble with six. He was convinced that no such pair of squares existed; in fact, he conjectured that if $n = 4k + 2$, k a positive integer, then no pair of orthogonal $n \times n$ orthogonal Latin squares exists.

In 1900, an English mathematician named Tarry showed that Euler was correct for the case $n = 6$. His more general conjecture, however, was not settled for 177 years when, in 1959, R. C. Bose and S. S. Shikhande constructed a pair of 22 × 22 orthogonal Latin squares! It is now known that there is a pair of orthogonal $n \times n$ Latin squares for every positive integer n except $n = 2$ and $n = 6$

Another type of combinatorial design, the *Hadamard matrix*, was used in constructing the code used in communicating with the Mariner 9, Voyager 1 and Voyager 2 spacecraft.

Results on these and other designs, which are studied in Pure Mathematics 4341, are still being discovered today. Combinatorial design theory is an excellent and exciting area in which to do research.

Note that a knowledge of some basic abstract algebra and linear algebra would be helpful in this course.

Text. One textbook used recently is *Combinatorics: A First Course* by A. P. Street and W. D. Wallis (Charles Babbage Research Centre).

Marks. Usually, 50% of the final grade is determined from assignments and 50% from a two hour final examination.

Calendar description. **4341 Combinatorial Designs** includes the study of finite fields, Latin squares, finite projective planes and balanced incomplete block designs.
Prerequisite: Mathematics 3320 or 3340.

Offered. Winter