MATHEMATICS 4321
GROUP THEORY

If each of the five faces of a regular triangular prism is to be painted with one of six available colours, how many essentially different colour arrangements (i.e., not obtainable from each other by rotations of the whole prism) are then possible?

Any sequence of rotational moves of a Rubik's cube rearranges the coloured faces of the individual cubelets, and the set of all obtainable such rearrangements forms a group under composition. What is the order of this group? (What would the answer be, if the standard Rubik's cube were replaced by one whose faces, instead of being uniformly coloured, carry six different pictures?)

Such enumeration problems and many other questions (such as *why is there no general formula for solving all fifth-degree polynomial equations?*) require for their solutions some further familiarity with the theory of groups, beyond what is covered in a first course in abstract algebra. So, this course is really a continuation of Mathematics 3320, and it also includes some additional material on fields and polynomial rings.

Text. Two books whose contents cover this course rather well are *Classical Abstract Algebra* by Richard A. Dean (Harper and Row) and the recent book *Introduction to Abstract Algebra* by W. K. Nicholson, PWS–Kent.

Marks. Typically, the final mark is based on a final examination (50%), a midterm test (30%) and on homework assignments (20%).

Calendar description. **4321 Group Theory** examines permutation groups, Sylow theorems, normal series, solvable groups, solvability of polynomials by radicals, introduction to group representations.
Prerequisite: Mathematics 3320.

Offered. Alternate Winters