MATHEMATICS 4320
RING THEORY

We are all familiar with sets of numbers such as the integers, the rationals and the reals. The addition and multiplication of numbers satisfy several rules; for example,

$$a + 0 = a, \quad a + b = b + a, \quad 1 \cdot a = a, \quad a(b + c) = ab + ac \quad \text{for any numbers } a,b,c$$

From these basic rules, one can deduce many interesting properties. One of the best known is that any positive integer can be written, in a unique way, as a product of prime numbers.

The known number systems can be studied simultaneously under the umbrella of one general theory. In this theory, one studies certain sets, called *rings*, equipped with two operations called addition and multiplication. In addition to the integers, rationals and reals, another example of a ring is the set of all polynomials with real coefficients.

In this course, some basic properties of rings are deduced and some special elements are introduced, such as *prime* elements. It is shown that in some rings (such as *principal ideal domains*}, every element can be written as a product of prime elements, but there are many more rings for which this property does not hold.

In the second part of the course, other algebraic objects called *modules* are studied. These are sets with an addition and a second operation that is multiplication of elements by ring elements. Examples of modules are abelian groups and vector spaces. Our attention will be mainly focused on determining the structure of modules over principal ideal domains.

Text. The following are suitable references:

1. B. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra* (Chapman and Hall).
2. D. J. H. Garling, *A Course in Galois Theory* (Cambridge University Press).
3. R. Y. Sharp, *Steps in Commutative Algebra* (Cambridge University Press).
4. L. H. Rowen, *Ring Theory* (Academic Press).

Marks. As with most senior courses, there is no established pattern for awarding marks. The exact distribution will be determined by the instructor.

Calendar description. **4320 Ring Theory** examines factorization in integral domains, structure of finitely generated modules over a principal ideal domain with application to Abelian groups, nilpotent ideals and idempotents, chain conditions, the Wedderburn-Artin theorem.
Prerequisite: Mathematics 3320.

Offered. Alternate Winters