

**MATHEMATICS 4300**  
**GENERAL TOPOLOGY**

Topology can be viewed in a number of ways, as a type of generalized geometry sometimes called *rubber sheet geometry*, or as the study of continuity, though without the  $\epsilon$  and  $\delta$  familiar from analysis. In Euclidean geometry, two triangles are congruent if one can be picked up and placed on top of the other so as to fit exactly. In topology, the concept analogous to *congruence* is that of *topological equivalence* or *homeomorphism*. This is a much looser relationship; any triangle is homeomorphic to any other triangle, to any square, and even to any circle! Intuitively, the basic idea is that if the triangle were made out of a sufficiently flexible piece of rubber, then it could be stretched into the shape of any other triangle, any square or any circle. The rules of this stretching allow all kinds of deformation but usually not cutting or gluing together. Technically, a *homeomorphism* is a bijective function (one-to-one correspondence),  $f$ , such that both  $f$  and its inverse  $f^{-1}$  are continuous. Hence we see the justification for looking at topology as the study of continuity.

Returning to the geometric viewpoint, we remark that a doughnut and a coffee cup are topologically equivalent! If a doughnut were made of a sufficiently flexible type of rubber, then it could be bent, squeezed, stretched and twisted into the shape of a coffee cup. An essential point of similarity here is, of course, that there is one hole through each object. On the other hand, a solid ball has no holes through it, and is not topologically equivalent to either a doughnut or a coffee cup. These considerations have prompted the joke that a topologist is someone who cannot tell the difference between a doughnut and a coffee cup!

**Text.** This course often proceeds without a formal text. The short book *Elements of General Topology* by Donald Bushaw (John Wiley) provides an excellent introduction to the subject. For years, *General Topology* by John L. Kelley (Van Nostrand) has been the standard textbook. Many books with *analysis* in the title have chapters on topology, for example, *Real Analysis* by H. L. Royden (MacMillan).

**Marks.** The marking scheme varies from instructor to instructor and from time to time.

**Calendar description.** **4300 General Topology** examines topological structure on a set, neighborhood, open and closed sets, continuity, sub-spaces and quotient spaces, connectedness, relation between topologies, base and sub-base, product spaces, applications to Euclidean spaces. Hausdorff, regular, normal and compact spaces, metric spaces, compacta and continua, metrizable. Prerequisite: Mathematics 3300, or Mathematics 3000.

**Offered.** Winter