MATHEMATICS 4230
DIFFERENTIAL GEOMETRY

Differential geometry, like all areas of calculus, displays two profound aspects:

(1) a means to solve practical problems, and
(2) an avenue to deep philosophic thought.

Classical differential geometry studies properties of curves and surfaces embedded in the three-dimensional Euclidean space in which we all live. The *geodesic problem*—to find the shortest distance between two points *P* and *Q* on a given surface—is one problem solved in differential geometry. (Pilots flying long distances fly along the geodesics for a sphere, the great circles.)

Differential geometry and its extensions by Riemann to *n*-dimensional *surfaces* happened to be exactly the type of mathematics Einstein needed to express his philosophic views concerning the universe. Using the mathematics of these disciplines, he was able to replace gravitational effects by a reformulation of space and time; i.e., the curved space-time of general relativity.

This course uses calculus and vectors to study the local properties of curves and surfaces, curvature being the most important.

Text. A large number of classical texts, most of them now out of print, exist in the library of reference books. One of the best of these is *Lectures on Classical Differential Geometry* by D. J. Struik. A recent book by Faber is also suitable, *Differential Geometry and Relativity Theory*, as is the book by Martin Lipschutz, *Differential Geometry*, in the *Schaum's Outline* series.

Marks. Generally 50% is assigned to a final examination, with 35% given to a one-hour term test and 15% to assignments.

Calendar description. **4230 Differential Geometry** covers theory of curves, Frenet relations, curvature and torsion, singular points of curves, first and second quadratic forms, classification of points on a surface, Gaussian curvature, Gauss-Weingarten theorem, Christoffel's symbols, theorema Egregium, Gauss-Codazzi-Mainardi theorem, internal geometry of surfaces, isometric and conformal mappings, geodesic curvature and torsion, parallel displacement, Gauss-Bonnet theorem.
Prerequisite: Mathematics 3202.

Offered. Alternate Winters