MATHEMATICS 4000
LEBESGUE INTEGRATION

The Riemann integral, introduced by Bernard Riemann around 1854, was known to have shortcomings as early as 1881. In that year, Vito Volterra gave an example of a function with a bounded derivative which was not Riemann integrable. This prompted Henri Lebesgue to begin looking for an improvement on Riemann's definition which he completed about 1902. Lebesgue's definition, which combined measure theory and integration, was an enormous step forward. His integral integrated more functions than Riemann's and sufficient conditions for interchanging the order of various limit processes were easier to check in practice. Lebesgue's integral is still the “default” integral in serious applications today.

Text. *Introduction to Topology and Modern Analysis* by George F. Simmons (McGraw-Hill) or a similar book. *Real Analysis: Courses Notes* by Bruce Watson is also available.

Marks. 20% for weekly assignments, 30% for midterm test and 50% for the final examination.

Calenda Description. 4000 Lebesgue Integration includes a review of the Riemann integral, functions of bounded variation, null sets and Lebesgue measure, the Cantor set, measurable sets and functions, the Lebesgue integral in $\mathbb{R}^1$ and $\mathbb{R}^2$, Fatou's lemma, Monotone and Dominated Convergence Theorems, Fubini's Theorem, an introduction to Lebesgue-Stieltjes measure and integration.
Prerequisite: Mathematics 3001
Note: Credit cannot be received for both of Mathematics 4000 and the former Pure Mathematics/ Statistics 4400.

Offered: Fall