

MATHEMATICS 3331
PROJECTIVE GEOMETRY

Everybody knows that the parabola, the ellipse, and the hyperbola are quadratic curves (the equation satisfied by the coordinates of their points is quadratic) but how do we know that these are the only quadratic curves? Such a question is very important in mathematics and in science. The way we study things, the way we try to understand things, is to classify those things and until the classification is complete, researchers are kept busy trying to find new objects that fit the classification. It turns out that there are nine quadratic curves, no more and no less! It is interesting that to get this simple answer you have to develop an entirely new kind of geometry and a formidable armoury of new geometric objects, such as points at infinity, as well as lines and planes at infinity. You also need a new type of coordinate system called *barycentric* coordinates. This new geometry is called projective geometry, and it was discovered by artists (the painters of the Renaissance) who were trying to establish the correct rules of perspective drawing, long before it attracted the attention of mathematicians.

Projective geometry unifies Euclidean geometry (which is at least 2,500 years old), affine geometry (about 250 years old) and the non-Euclidean (hyperbolic and elliptic) geometries discovered in the 19th century. It does this by stripping other geometries of their metric structure (length, angle, perpendicularity) or their affine structure (ratio, orientation, parallelism), only to restore these structures piecemeal. The stripping process does not denude geometry completely; interesting structures such as collinearity, cross-ratio and incidence remain. In this way, a hierarchy of geometries arises showing the breadth and depth of the geometric landscape. For thousands of years geometry has been the paragon of science. Scientists working in other fields have been vying with one another in trying to shape their own discipline more *geometrico*, that is, *in the pattern of geometry*. With the proliferation of disciplines and in the pursuit of specialization this fact may have been obscured in our own century, but there is no doubt that geometry, and in particular, projective geometry, permeates scientific and philosophical thinking.

Text. There are many excellent books on projective geometry, including several by the great Canadian geometer H. S. M. Coxeter (*Introduction to Geometry, Non-Euclidean Geometry, The Real Projective Plane*).

Marks. Typically, 50% of the evaluation credit is awarded for the final examination, while 25% for the midterm test and 25% for the home assignments.

It is recommended that the students interested in geometry take the courses 3330 and 3331 in numerical order.

Calendar description. **3331 Projective Geometry** includes course topics: projective space, the principle of duality, mappings in projective space, conics and quadrics.
Prerequisite: Mathematics 2051 or 2320.

Offered. Winter