MATHEMATICS 3303
INTRODUCTORY GEOMETRIC TOPOLOGY

Topology is a type of geometry, not of points, lines and triangles but rather of surfaces, higher dimensional analogues of surfaces and other shapes.

In this course, we study the geometry of surfaces such as the plane, the sphere, the Mobius strip, the torus, the Klein bottle, real projective space, and spheres with handles, and also of knots such as the square knot, the granny knot, the trefoil knot and the figure of eight knot. We also investigate the geometry of graphs; i.e., configurations of points joined by arcs.

Our techniques include use of the Euler characteristic of a surface, a number associated with a surface (for the torus it is 0, for real projective space 1, for the sphere 2, etc.) that gives some deep insight into the geometry of that surface. We also use the fundamental group, a group associated with shapes such as surfaces and knots (for the torus it is an infinite group, for real projective space a group with two elements, for the sphere it is the zero group), which again incorporates crucial information concerning the geometric configuration that is being discussed.

Famous results considered include the proof that there are just five Platonic solids (tetrahedron, cube, octahedron, dodecahedron and icosahedron), and the map colouring problem. Surprisingly, determination of the number of colours needed can be relatively easy for surfaces other than the plane (it is incredibly difficult, and far beyond the scope of an undergraduate course, to show that four colours suffice in the plane; yet we will give an easy proof that seven colours suffice for the torus).

Our approach is in the first instance intuitive, although we later introduce the language of point-set topology to give a formal and precise interpretation of the concepts and results considered. The course does not assume any previous knowledge of group theory.

Text. There does not seem to be any book that is completely satisfactory for this course. *An Introduction to Topology and Homotopy* by A. J. Sieradski includes material on all of our topics and much more. *Intuitive Concepts of Topology* by B. H. Arnold covers most of our course content, at approximately the right level of difficulty, but does not discuss either the fundamental group or knot theory. A complete set of notes is given.

Marks. A typical breakdown would be 15% for homework, 25% or a midterm test and 60% for the final examination.

Calendar description. **3303 Introductory Geometric Topology** covers graphs and the four colour problem, orientable and non-orientable surfaces, triangulation, Euler characteristic, classification and colouring of compact surfaces, basic point-set topology, the fundamental group, including the fundamental groups of surfaces, knots, and the Wirtinger presentation of the knot group.
Prerequisite: Mathematics 2320.

Offered. Occasionally