MATHEMATICS 3300
SET THEORY

This course has a somewhat more philosophical orientation then other mathematics offerings. The following are some interesting examples of topics discussed.

1. Consider the following problem:

   In a certain town there is a barber (a man) who shaves all of the men who do not shave themselves, and these are the only men whom he shaves. We then ask who shaves the barber?

   If he does not shave himself then it follows that he does shave himself. If he does shave himself then he doesn't shave himself. So there seems to be no answer to the question!

2. Russell's paradox, as stated below, is similar to the barber argument, but it is also much more important. In fact it has major consequences for the way that we formulate set theory, and hence for the foundations of mathematics. We have to consider sets which are not elements of themselves. (The converse, sets which are elements of themselves, is a strange idea. Can you think of an example of such a set?)

   Russell's argument goes as follows.

   Let S be the set of all sets which are not elements of themselves. Here's the question: Is S an element of itself?

   The trouble is that if $S \in S$ then, by the definition of S, $S \notin S$, and if $S \notin S$ then $S \in S$ by similar reasoning. So we are left with a paradox that could undermine the logical foundation of mathematics!

3. The set of all natural members and the set of all real numbers are infinite sets, and the numbers of elements in each of these sets are infinite numbers (technically *cardinal* numbers). It is not difficult to prove that there is no one-to-one correspondence between the set of all natural numbers and the set of all real numbers, so the number of real numbers is a bigger infinity than the number of natural numbers. In Mathematics 3300 we discuss the theory of infinite cardinal numbers, including topics such as how they can be added together and multiplied together, and whether or not there is such a thing as the largest infinite cardinal number.

Text. *Set Theory with Applications* by S-Y Lin and Y-F Lin or a similar book.

Marks. It would be typical to assign 60% to the final exam, 25% to one mid-term test and 15% to homework.

Calendar description. **3300 Set Theory** is an introduction to Mathematical Logic, functions, equivalence relations, equipotence of sets, finite and infinite sets, countable and uncountable sets, Cantor's Theorem, Schroeder-Bernstein Theorem, ordered sets, introduction to cardinal and ordinal numbers, logical paradoxes, the axiom of choice.
Prerequisite: Mathematics 2320.

Offered. Fall