

MATHEMATICS 3132
NUMERICAL ANALYSIS I

The equation $x^3 + 4x^2 - 10 = 0$ is known to have a solution in the interval $[1, 2]$. Three people look at this problem and each one notices that with varying degrees of algebra the equation can be expressed in the form $x = g(x)$. Some of the possibilities are:

$$x = x - x^3 - 4x^2 + 10, \quad x = \frac{1}{2}\sqrt{10 - x^3}, \quad x = x - \frac{x^3 + 4x^2 - 10}{3x^2 + 8x}$$

The form $x = g(x)$ suggests the use of the following scheme: choose an initial guess x_0 , and then generate successive estimates via the iteration scheme $x_{k+1} = g(x_k)$, $k = 0, 1, \dots$. With $x_0 = 1.5$, this scheme is applied to the three functional relations above and the results are:

For the first formula, the numbers x_k diverge.

For the second formula, the numbers x_k converge to the root 1.365230013 in 30 steps.

For the third formula, the numbers x_k converge to the root 1.365230013 in four steps.

Although each method attempts to solve the same problem, they perform very differently within the context of the problem at hand. Why does the first method fail? Why does the third method work so much better than the second?

In numerical analysis, we not only examine numerical methods, we also quantify the appropriateness of problems for specific numerical methods. This will include general convergence criteria, error estimates with the consideration of computational costs, and a look at practical issues relating to implementation of specific methods.

Text. In recent years the text for this course has been *Numerical Analysis* by Richard Burden and J. Douglas Faires.

Marks. While the exact formula may vary from semester to semester, it is typical to assign 50% of the marks to term work and the remaining 50% to a final exam.

Calendar description. **3132 Numerical Analysis I** includes a discussion of round-off error, the solution of linear systems, iterative methods for nonlinear equations, interpolation and polynomial approximation, least squares approximation, fast Fourier transform, numerical differentiation and integration. There will be a 90 minute laboratory session each week.

Prerequisite: Math 2000, Math 2050, and a computing course (Computer Science 1510 is recommended).

Note: Credit cannot be obtained for both Mathematics 3132 and Computer Science 3731.

Offered. Fall