MATHEMATICS 3000
REAL ANALYSIS I

An interesting method for calculating $\sqrt{2}$, known in Mesopotamia before 1500 BC, can be described as follows:

Let $x_1 > 0$ be arbitrary, and for $n \geq 1$, define $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$. As n gets *large enough*, the term x_n gets *very close* to $\sqrt{2}$. In other words, the sequence $\{x_n\}$ *converges* to the real number $\sqrt{2}$.

This is a typical instance of a *limiting process*. This course is an introduction to the real line (the real number system), and certain limiting processes therein. This material forms the heart of all mathematics, and in its present state of development, brings to fruition more than two thousand years of mathematical work in geometry and algebra. As such, it is an *essential part* of the training of any student of mathematics, pure or applied. Having been exposed to a year or so of calculus as a working tool, where statements like $0 < 1$, $\lim_{n\to\infty} 1/n = 0$, and $\lim_{x\to 2} x^2 = 4$ are taken for granted without a formal proof, students will learn to ask the question (WHY?) for each statement made, and will learn the concepts in a more rigorous and formal way. In other words, they will be able to make precise mathematical statement in an unambiguous manner, and in this process will learn the so-called art of mathematical sophistication.

Proofs in mathematics are not merely a bunch of statements put together to draw up a desired conclusion, rather they are cleverly and delicately woven fabrics of thought, to be understood as a single theme. Students will be trained to accept statements, only if they can support them with valid and rigorous proofs.

Completeness of the real number system is the central theme of this course, and this will be glorified in various forms like the supremum principle, convergence of Cauchy sequences, the Bolzano- Weierstrass property, the ideas of lim sup, lim inf and the Heine Borel covering property. Most of the basic notions of calculus will be revisited in a more rigorous fashion.

Text. The following books have each been used in the recent past: *Foundations of Analysis: The Theory of Limits* by H. S. Gaskill and P. P. Narayanaswami (Harper and Row) (course content approximately chapters 0-4), and *Introductory Analysis: The Theory of Calculus* by J. A. Fridy (Harcourt Brace Jovanovich). *Real Analysis: Courses Notes* by Bruce Watson is also available.

Marks. Usually, 20% for weekly assignments, 30% for a one-hour mid-term examination and 50% for a comprehensive final examination at the end of the semester.

Calendar description. **3000 Real Analysis I** covers proof techniques, structure of $\mathbb{R}$, sequences, limits, continuity, uniform continuity, differentiation. Three lecture hours and one 90-minute laboratory per week. Prerequisite: Mathematics 2000.
Note: Credit can be obtained for only one of Mathematics 3000 and the former Mathematics 2001.

Offered. Fall and Winter