MATHEMATICS 2051
LINEAR ALGEBRA II

This course is a continuation of Mathematics 2050. Whereas in Mathematics 2050, the emphasis is on Euclidean *n*-space ($\mathbb{R}^n$) and linear transformations $\mathbb{R}^n \to \mathbb{R}^m$, in Mathematics 2051 we study finite-dimensional vector spaces and linear transformations from vector space to another. (A *vector space* is a natural extension of the concept of Euclidean *n*-space.)

A *diagonal* matrix is a matrix that is all zeros except on the main diagonal; that is, a matrix of the form

$$\begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix}$$

where the entries not shown are zero. A square matrix A is *diagonalizable* if there is an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. The vector space setting permits a beautiful and natural way to view diagonalizability.

The ability to diagonalize a matrix is quite fundamental in business, biology, engineering, economics and the social sciences. Suppose that the populations of three regions A, B and C shift each year according to the following rules: each year $\frac{1}{4}$ of the residents of A and $\frac{1}{4}$ of the residents of B move to C; $\frac{1}{6}$ of those in C move to A and $\frac{1}{3}$ of those in C move to B. Assuming an initial total population of 10 000, what is the long term distribution of the population, and does the answer depend on the initial distribution? Ideas from Mathematics 2051—eigenvalue, eigenvector, diagonalization—can be used to solve this problem!

Text. The text for this course has been, at various times, *Linear Algebra: A Pure and Applied First Course* by Edgar Goodaire (Pearson) **OR** *Elementary Linear Algebra* by Howard Anton (Wiley) **OR** *Linear Algebra with Applications* by Keith Nicholson (McGraw Hill).

Marks. Although the exact formula may vary from semester to semester, normally 40% of the final grade is given for assignments and a term test, and 60% for a final examination.

Calendar description. **2051 Linear Algebra II** includes the topics: real and complex vector spaces, basis, dimension, change of basis, eigenvectors, inner products, and diagonalization of Hermitian matrices. Prerequisite: Mathematics 1000 and 2050.

Offered. Fall and Winter