

Syllabus for the Qualifying Review in ANALYSIS

The examination will be based on the following topics:

1. Real Analysis: Properties of real numbers. Completeness (various approaches). Sequences, monotone sequences, convergence, limit superior and limit inferior. Functions and their limits, continuity, properties of continuous functions defined on a compact set. Uniform continuity. Infinite series, tests for convergence, power series, Taylor series, Taylor remainder. Differential calculus, mean value, theorems. Riemann integral, uniform convergence of sequences and series, Weierstrass approximation theorem. Transcendental functions. Elementary ideas in Fourier series. Functions of several variables; limits, continuity, partial derivatives and differentiability. Inverse function theorem. Implicit function theorem. Metric spaces: Cauchy sequences, completeness, completion of a metric space, uniform continuity, subspaces, Compactness.
2. Lebesgue Theory: Set functions, σ -fields, construction of Lebesgue measure, measurable functions, Lebesgue integral, comparison with Riemann integral, convergence theorems, Fubini's theorem for \mathbb{R}^2 .
3. Complex Analysis: Elementary analytic functions, characterization of analyticity, harmonic functions, Cauchy's theorem for contour integrals, power series, Laurent series, isolated singularities, the residue theorem; calculus of residues.
4. Functional Analysis: Elementary theory of normed vector spaces, Banach spaces and Hilbert spaces; linear operators on Banach and Hilbert spaces; compact operators.

References:

- H. L. Royden. Real Analysis (Chapters 3 to 5).
- R. V. Churchill and J. W. Brown. Complex Variables and Applications (Chapters 1 to 7).
- W. Rudin. Principles of Mathematical Analysis.
- G. De Barra. Measure Theory and Integration (Chapters 1 to 5).
- M. H. Protter and C. B. Morrey (1991). A First Course in Real Analysis, Second edition. Springer.
- L. Debnath and P. Mikusinski (1990). Introduction to Hilbert Spaces with Applications. Academic Press.