

Syllabus for the Qualifying Review in ALGEBRA

The examination will be based on the following topics:

1. Linear algebra: vector spaces, subspaces, quotient spaces, dual spaces, linear transformations, matrices, change of bases, rank and nullity, determinants, eigenvalues and eigenvectors, rational and Jordan forms of a matrix, inner product spaces, diagonalization of self-adjoint transformations and Hermitian forms.
2. Groups: subgroups, Lagrange's theorem, homomorphisms, normal subgroups, quotient groups, isomorphism theorems for groups, direct products, fundamental theorem on finitely generated abelian groups, symmetric groups and alternating groups, group actions, Sylow theorems, automorphisms, composition series, Jordan-Holder-Schreier theorem, nilpotent groups, solvable groups.
3. Fields: subfields, isomorphisms, algebraic and transcendental extensions, separable and inseparable extensions, splitting fields, fundamental theorem of Galois theory, finite fields, algebraically closed fields.
4. Rings: subrings, homomorphisms, ideals, quotient rings, direct products, matrix rings, polynomial rings, Jacobson radical, Artinian and Noetherian rings, Wedderburn-Artin theorem, rings of quotients of an integral domain, prime ideals, maximal ideals, Euclidean domains, principal ideal domains, unique factorization domains.
5. Modules: submodules, quotient modules, isomorphism theorems for modules, free modules, projective and injective modules, simple and semisimple modules, structure of finitely generated modules over principal ideal domains.
6. Introduction to representations of finite groups: irreducible and completely reducible representations, Maschke's theorem, regular representation, characters.

References:

- J. A. Beachy, Introductory lectures on rings and modules, *London Math. Soc. Student Texts* **47**, Cambridge University Press, Cambridge, 1999.
- I. N. Herstein, Topics in Algebra, John Wiley, New York, 1975.
- K. Hoffman and R. Kunze, Linear algebra, second edition, Prentice-Hall, Englewood Cliffs, N.J., 1971.
- W. K. Nicholson, Introduction to abstract algebra, third edition, Wiley-Interscience, New Jersey 2007.

Additional References:

- Y. A. Bahturin, Basic structures of modern algebra, *Mathematics and its Applications* **265**, Kluwer Academic Publishers Group, Dordrecht, 1993.
- S. Lang, Algebra, Revised third ed. *Graduate Texts in Mathematics* **211**, Springer-Verlag, New York, 2002.
- W. K. Nicholson, Linear Algebra with Applications, Fifth ed. McGraw-Hill Publishing, 2006. (especially Chapters 6-10)