MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

PhD Comprehensive Exam | **Statistics Exam I** | Summer 2022

Last Name: ______________ First Name: ______________ Student No: ______________

Set by Dr. H. Wang

OFFICE USE ONLY

Grade: ______________

Instructions:

1. Please following these instructions carefully.
2. This is a closed-book exam.
3. Please write your name in the designated areas.
4. Please write your student number in the designated area.
5. All questions/sub-questions are equally weighted.
6. You have 120 minutes to complete this exam.
7. Computer package R and a calculator are allowed during the exam.

1. In a group of 20 people there are three brothers. The group is separated at random into two subgroups G_1 and G_2 with equal size of 10 people in each subgroup. What is the probability that the brothers are in the same subgroup?

2. Assuming each year has 365 days,

   (a) what is the probability that each person has a different birthday in a group of $n = 40$? (keep 4 digits after the decimal point)

   (b) find the largest n, the number of people in a group, such that, the probability that out of n people all have different birthdays is greater than 0.50.

   (c) if n varies, the probability that out of n people all have different birthdays can be considered as a function of n. Argue about the monotonicity of this function.

3. (The Miner Problem) Consider a man trapped in a room that contains 3 doors. Door 1 leads him to a safe place after 1-days' travel; door 2 returns him back to the same room after 2-days' journey; and door 3 returns him back to the same room after 5-days' journey. Suppose at all times he is equally-likely to choose any of the three doors, and let X denote the time it takes the miner to a safe place. Find $E[X]$.

4. Suppose that X and Y are independent and both have a standard normal distribution. Find the PDF of the quotient $U = \dfrac{X}{Y}$.

5. (The Hats Problem) Suppose that n people have their hats returned at random. Let $X_i = 1$ if the ith person gets his or her own hat back and 0 otherwise. Let $S_n = \sum_{i=1}^{n} X_i$. Then S_n is the total number of people who get their own hats back. Show that

   (a) $E[S_n] = 1$ and $Var(S_n) = 1$.

   (b) $P(S_n \geq 11) \leq 0.01$ for any $n \geq 11$.

6. Consider a sequence of i.i.d. random variables $X_1, X_2, \ldots$ from a common distribution with PDF:

$$f_X(x) = \frac{3}{2}x^2, \quad \text{for} \quad -1 \leq x \leq 1.$$

   (a) Find the limiting distribution of the minimum of $\{X_1, X_2, \ldots, X_n\}$ as $n \to \infty$.

   (b) Define the partial sum of $X_1, X_2, \ldots$ by

$$S_n = X_1 + X_2 + \cdots + X_n.$$

   Find the approximated probability $P(S_{100} \leq 2.50)$.