PhD Comprehensive examination in Numerical Analysis

Department of Mathematics and Statistics
Memorial University.

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1. **Basic Numerical Analysis**: Floating point arithmetic. Polynomial and piecewise polynomial interpolation, splines, best approximation. Basic and Gaussian quadrature. Numerical differentiation and Richardson extrapolation. Fourier series and fast Fourier transforms.

2. **Linear and nonlinear algebraic equations**: Matrix and vector norms, conditioning, sensitivity. Direct methods: Gaussian elimination, including LU and Cholesky factorizations. Iterative methods for linear systems: Gauss-Seidel, Jacobi, successive over-relaxation. M-matrices, positive-definite matrices. Solution of linear least squares problems, including normal equations and QR factorization. Fixed points, contraction mapping theorem, error estimates. Newton's method for nonlinear systems. Unconstrained optimization, including Newton's method, steepest descent, and line search methods.

3. **Numerical methods for ODEs**: Explicit and implicit methods for ODEs: one–step and multistep methods, e.g. Euler, Leap-Frog, Runge-Kutta, Adams-Bashforth/Moulton methods. Stability regions in the complex plane. Truncation, local and global error.

4. **Numerical methods for PDEs**: Elliptic PDEs with Dirichlet, Neumann, and Periodic boundary conditions. Basic concepts of collocation and Galerkin methods; finite difference and finite element methods for elliptic PDEs. Fast Poisson solvers. Finite difference methods for parabolic PDEs. Finite difference and finite volume methods for hyperbolic PDEs. Stability, consistency, convergence, and the Lax equivalence theorems for steady-state and time-dependent PDEs. von Neumann and modified PDE analysis. Spectral properties of finite difference operators. Courant–Friedrichs–Lewy conditions.

Recommended readings:

- U. Ascher and C. Greif, A First Course in Numerical Methods, SIAM, 2011
- R. L. Burden, J. D. Faires, and A. M. Burden, Numerical Analysis, 10th ed. Brooks/Cole, 2016.

- D. Kincaid and W. Cheney, Numerical Analysis: Mathematics of Scientific Computing, 3rd ed., Brooks/Cole, 2002.
- R. J. LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations, SIAM, 2007.

Supplementary readings:

- M. L. Overton, Numerical Computing with IEEE Floating Point Arithmetic, SIAM, 2001.
- J. W. Demmel, Applied Numerical Linear Algebra, SIAM, 1997.
- K. W. Morton and D. F. Mayers, Numerical Solution of Partial Differential Equations, 2nd ed., Cambridge, 2005.
- A. Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge, 2008.