Departmental Colloquium

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Friday, March 31, 2017
2:00-3:00 pm in HH-3017

Slicing inequalities for measures of convex bodies

Abstract:

We consider the following problem. Does there exist an absolute constant C such that for every $n \in N$, every integer $1 \leq k < n$, every origin-symmetric convex body L in R^n, and every measure μ with non-negative even continuous density in R^n,

$$\mu(L) \ \leq \ C^k \max_{H \in Gr_{n-k}} \mu(L \cap H) \, |L|^{k/n}, \tag{1}$$

where Gr_{n-k} is the Grassmanian of $(n-k)$-dimensional subspaces of R^n, and $|L|$ stands for volume? This question is an extension to arbitrary measures (in place of volume) and to sections of arbitrary codimension k of the hyperplane conjecture of Bourgain, a major open problem in convex geometry.

We show that (1) holds for arbitrary origin-symmetric convex bodies, all k and all μ with $C \sim \sqrt{n}$, and with an absolute constant C for some special classes of bodies, including unconditional bodies, unit balls of subspaces of L_p, and others. We also prove that for every $\lambda \in (0,1)$ there exists a constant $C = C(\lambda)$ so that inequality (1) holds for every $n \in N$, every origin-symmetric convex body L in R^n, every measure μ with continuous density and the codimension of sections $k \geq \lambda n$. The latter result is new even in the case of volume.

The proofs are based on a stability result for generalized intersection bodies and on estimates of the outer volume ratio distance from an arbitrary convex body t o the classes of generalized intersection bodies.