Thesis Seminar

Speaker

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Local Fourier Analysis for Saddle-Point Problems

Abstract:

The numerical solution of saddle-point problems has attracted considerable interest in recent years, due to their indefiniteness and often poor spectral properties that make efficient solution difficult. While much research already exists, developing efficient algorithms remains challenging. Researchers have applied finite-difference, finite-element, and finite-volume approaches successfully to discretize saddle-point problems, and block preconditioners and monolithic multigrid methods have been proposed for the resulting systems. However, there is still much to understand.

Magnetohydrodynamics (MHD) models the flow of a charged fluid, or plasma, in the presence of electromagnetic fields. Often, the discretization and linearization of MHD leads to a saddle-point system. We present vector-potential formulations of MHD and a theoretical analysis of the existence and uniqueness of solutions of both the continuum two-dimensional resistive MHD model and its discretization.

Local Fourier analysis (LFA) is a commonly used tool for the analysis of multigrid and other multilevel algorithms. We first adapt LFA to analyse the properties of multigrid methods for both finite-difference and finite-element discretizations of the Stokes equations, leading to saddle-point systems. Monolithic multigrid methods, based on distributive, Braess-Sarazin, and Uzawa relaxation are discussed. From this LFA, optimal parameters are proposed for these multigrid solvers. Numerical experiments are presented to validate our theoretical results. A modified two-level LFA is proposed for high-order finite-element methods for the Laplace problem, curing the failure of classical LFA smoothing analysis in this setting and providing a reliable way to estimate actual multigrid performance. Finally, we extend LFA to analyze the balancing domain decomposition by constraints (BDDC) algorithm, using a new choice of basis for the space of Fourier harmonics that greatly simplifies the application of LFA. Improved performance is obtained for some two- and three-level variants.