Functional Identities
and
Zero Product Determined Algebras

Mini course by

Professor

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A *functional identity* is an identical relation in a ring R that, besides ring elements, also involves functions from R^n to R which are considered as unknowns. The goal is to find their forms. While this can be accomplished in rather general rings, the problem becomes, apparently paradoxically, very difficult in some well understood classes of rings such as PI rings of low degree. An algebra A over a field F is said to be *zero product determined* if for every bilinear map $f: A \times A \to F$ the property that $ab = 0$ implies $f(a, b) = 0$ there exists a linear functional φ on A such that $f(a, b) = \varphi(ab)$ for all $a, b \in A$.

Here, A may be any nonassociative algebra; most results, however, concern associative and Lie algebras. As a sample result, we mention that a finite-dimensional associative unital algebra is zero product determined if and only if it is generated by idempotents. Results on functional identities have turned out to be applicable to various problems in noncommutative algebra, nonassociative algebra, linear algebra, and operator theory. In particular, they have been used as an essential tool for solving Herstein's Lie map conjectures.

Results on zero product determined algebras also have a variety of applications, especially in the theory of Banach algebras. Although the two theories, that of functional identities and that of zero product determined algebras, are quite different, they both arose from similar questions and, moreover, have some related applications. This makes it possible to consider them concurrently in this mini course.

Matej Brešar is Professor of Mathematics at University of Ljubljana and University of Maribor, Slovenia. He is the author or co-author of over 150 research papers, the co-author of the book *Functional Identities* (Birkhäuser, 2007), and the author of the book *Introduction to Noncommutative Algebra* (Springer, 2014). According to MathSciNet, his works have been cited over 3000 times. He is an Associate Member of Slovenian Academy of Sciences and Arts.

Mini course schedule:

Monday Sept. 10, Tuesday Sept. 11, Wednesday Sept. 12, Thursday Sept. 13, Friday Sept. 14

12:00 noon - 12:50 pm

HH-3017