

# Colloquium

Dr. Shu Zhang  
Tongji University, China

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## *Synchronization and Desynchronization Induced by Time Delay in an Internet Congestion Control Model*

### **Abstract:**

Congestion, or traffic jam, is among the most stimulating problems of the Internet science. This phenomenon may occur when the demand of service goes beyond the capability of processing provided by the network. Various dynamical models, which are expressed in terms of differential equations, are established to describe the evolution, especially the oscillation, of the state variable of the network, namely, the speed of data transmission. Among all the types of oscillations, complete synchronization attracts most attention since the risk of congestion is increased significantly when the speed of data transmission of each connection approaches the maximum simultaneously. Both theoretical and experimental researches show that time delay plays a crucial role in the occurrence of oscillation of the speed of data transmission. In the present work, we will study the effect of time delay on the synchronization of the Internet congestion control model and provide a scheme of desynchronization based on the state feedback control.

We first consider an n-dimensional congestion control model with ring topology is considered to study the synchronous oscillation induced by the time delay via the Hopf bifurcation. The expression of the possible critical delay for the Hopf bifurcation is obtained by analyzing the linearized system around the equilibrium. Then the method of multiple scales is employed to obtain the quantitative relation between the delay and the induced synchronous oscillation. The analytical results agree well with those obtained by numerical method. Furthermore, the effects of other parameters are studied either. Second, for a congestion control model of arbitrary dimension with star-type topology, we propose a desynchronization scheme by introducing state feedback controllers in such a way that the sum of all the components of the eigenvector of the linearized system around the equilibrium is zero. By means of the method of multiple scales, the conditions on the values of controller parameters such that the desynchronous oscillation arises are obtained and two different patterns of desynchronization, i.e., discrete wave and antisynchronization are observed, depending on the parity of number of connections. Numerical simulations are carried out to validate the theoretical analysis and the effectiveness of the propose scheme is confirmed. This is a joint work with Dr. Kwok-wai Chung in City University of Hongkong.